

1. Answers may vary. When a line intersects a circle, if it intersects at only one point, then it forms a right angle with the radius of the circle to that point. If it intersects at two points, it forms a chord of the circle, which is perpendicular to the radius that passes through its midpoint. When a pair of lines intersect a circle, they form an angle whose measure depends on the intercepted arcs of the circle. The lengths of the segments formed are determined by the distance from the point of intersection to each intersection point with the circle.
2. A **sector of a circle** is a region of a circle with two radii and an arc of the circle as borders.
3. An **inscribed angle** is an angle with its vertex on the circle.
4. A **tangent to a circle** and the corresponding circle have exactly one point in common.
5. When both rays of an angle intersect a circle, the **intercepted arc** is the portion of the circle between the rays.
6. Use the formula $s = \frac{n}{360}(2\pi r)$ where n is the measure of the central angle and s is the arc length to find \widehat{JK} .

$$s = \frac{112}{360}(2\pi \cdot 5)$$

$$s = \frac{28}{9}\pi$$

7. Use the formula $s = \frac{n}{360}(2\pi r)$ where n is the measure of the central angle and s is the arc length to find \widehat{ABC} .

$$n = 360^\circ - 154^\circ = 206^\circ$$

$$s = \frac{206}{360}(2\pi \cdot 8)$$

$$s = \frac{412}{45}\pi$$

8. Write the formula for the area of a sector, and solve using the given values of the measure, n , of the central angle and the radius r .

$$A = \frac{n}{360} \pi r^2$$

$$A = \frac{63}{360} \pi (10)^2$$

$$A = \frac{35}{2} \pi$$

9. Write the formula for the area of a sector, and solve using the given values of the measure, n , of the central angle and the radius r .

$$A = \frac{n}{360} \pi r^2$$

$$= \frac{172}{360} \pi (7)^2$$

$$= \frac{2,107}{90} \pi$$

10. From the circumference, you can calculate the radius. Since the area is determined by the radius and central angle, if you know the area and radius, you can set up an equation for the sector area and solve for the central angle.

11. $\overline{PQ} \perp \overline{QR}$ by Theorem 10-1. Therefore, $\triangle PQR$ is a right triangle. $PQ = 7$ because it is a radius of $\odot P$. Use the Pythagorean Theorem to find QR .

$$7^2 + QR^2 = (7 + 6)^2$$

$$QR^2 = 13^2 - 7^2$$

$$QR = \sqrt{169 - 49}$$

$$QR = 2\sqrt{30}$$

12. Since \overline{CB} is a radius and \overline{AB} is tangent to $\odot P$, $\overline{CB} \perp \overline{AB}$ by Theorem 10-1. Therefore, $m\angle ABC = 90^\circ$. Use the Triangle Angle-Sum Theorem to solve for $m\angle CAB$.

$$m\angle CAB + m\angle BCA + m\angle ABC = 180^\circ$$

$$m\angle CAB = 180^\circ - m\angle BCA - m\angle ABC$$

$$m\angle CAB = 180^\circ - 58^\circ - 90^\circ$$

$$m\angle CAB = 32^\circ$$

13. no; The tangents are both perpendicular to the diameter, so they are parallel to each other and never intersect.

14. Since \overline{MN} is tangent to $\odot L$, $\angle MNL = 90^\circ$. Therefore, $\triangle MNL$ is a right triangle and the Pythagorean theorem can be used to solve for x .

$$\begin{aligned}x^2 + 8^2 &= (5 + x)^2 \\x^2 + 64 &= x^2 + 10x + 25 \\39 &= 10x \\x &= 3.9\end{aligned}$$

The radius is $x = 3.9$ cm.

15. We know that $r = 7$. The diagram indicates that $\overline{AB} \cong \overline{CD}$ and $TE = 5$. Therefore, by the Converse of Theorem 10-5, $\overline{ET} \cong \overline{TF}$. Since \overline{TH} is a radius of $\odot T$, $TH = 7$. Solve for FH .

$$\begin{aligned}TH &= TF + FH \\FH &= TH - TF \\&= 7 - 5 \\&= 2\end{aligned}$$

16. The diagram indicates that $CF = FD$ and \overline{TH} is a radius of $\odot T$. Therefore, by the Converse of Theorem 10-6, $\overline{TH} \perp \overline{CD}$. Also, $\overline{AB} \cong \overline{CD}$ and $ET = 5$. Therefore, by the Converse of Theorem 10-5, $TF = 5$. $\triangle TFC$ is a right triangle. Use the Pythagorean Theorem to solve for CF .

$$\begin{aligned}CF^2 + TF^2 &= CT^2 \\CF^2 &= CT^2 - TF^2 \\CF &= \sqrt{CT^2 - TF^2} \\CF &= \sqrt{7^2 - 5^2} \\CF &= \sqrt{49 - 25} \\CF &= \sqrt{24} \\CF &\approx 4.9\end{aligned}$$

$CD = CF + FD$ and $CF = FD$. Therefore, $CD = 4.9 + 4.9 = 9.8$.

17. The diagram indicates that $CF = FC = BE = AE$ and $m\angle CTH = 44^\circ$. By Theorem 10-3 $m\angle CTH = m\angle BTG = m\angle GTA = 44^\circ$. So $m\angle BTA = 44^\circ + 44^\circ = 88^\circ$.

18. They are perpendicular. Let X be the midpoint of \overline{AB} . Then $\overline{TX} \perp \overline{AB}$ and $\overline{SX} \perp \overline{AB}$. So, T , X , and S are collinear and $\overleftrightarrow{TS} \perp \overline{AB}$.

19. The cut creates a horizontal chord. The diameter of the countertop is perpendicular to the chord. By Theorem 10-6, the diameter bisects the chord. The diameter of the circle is 120 cm, so the radius is 60 cm. The distance between the center of the countertop and the horizontal chord is $100 - 60 = 40$ cm. Half of the chord, a radius, and a line segment from the center to the chord form a right triangle. Use the Pythagorean Theorem to solve for the chord length.

$$\left(\frac{1}{2}x\right)^2 + 40^2 = 60^2$$

$$\frac{1}{4}x^2 = 60^2 - 40^2$$

$$x^2 = 4(60^2 - 40^2)$$

$$x = \sqrt{4(60^2 - 40^2)}$$

$$x \approx 89.4$$

The length of the cut is 89.4 cm.

20. $\angle EDF$ and $\angle EGF$ both intercept \widehat{EF} . Therefore, by Corollary 1, $\angle EDF \cong \angle EGF$. Set the angle measurements equal and solve for x .

$$m\angle EDF = m\angle EGF$$

$$5x + 14 = 9x - 6$$

$$20 = 4x$$

$$x = 5$$

Use Theorem 10-8 to write an equation and solve for $m\widehat{EF}$.

$$m\angle EDF = \frac{1}{2}m\widehat{EF}$$

$$m\widehat{EF} = 2m\angle EDF$$

$$= 2(5(5) + 14)^\circ$$

$$= 2(25 + 14)^\circ$$

$$= 2(39)^\circ$$

$$= 78^\circ$$

21. Use Theorem 10-9 to find $m\widehat{NP}$.

$$m\angle NP = \frac{1}{2}m\widehat{NP}$$

$$m\widehat{NP} = 2m\angle NP$$

$$= 2(51^\circ)$$

$$= 102^\circ$$

The arcs of a circle add up to 360° . Use this fact to find $m\widehat{MP}$.

$$m\widehat{MP} + m\widehat{NP} + m\widehat{MN} = 360^\circ$$

$$m\widehat{MP} = 360^\circ - m\widehat{NP} - m\widehat{MN}$$

$$= 360^\circ - 102^\circ - 163^\circ$$

$$= 95^\circ$$

Use Theorem 10-8 to find $m\angle MNP$.

$$m\angle MNP = \frac{1}{2}m\widehat{MP}$$

$$= \frac{1}{2}(95^\circ)$$

$$= 47.5^\circ$$

22. Corollary 3 states that opposite angles in an inscribed quadrilateral are supplementary.

This gives the following equations.

$$\begin{aligned}m\angle ZWX + m\angle XYZ &= (6x)^\circ + (9x)^\circ = 180^\circ \\m\angle WZY + m\angle YXW &= m\angle WZY + (8x + 5)^\circ = 180^\circ\end{aligned}$$

Solve the first equation for x .

$$\begin{aligned}6x + 9x &= 180 \\15x &= 180 \\x &= 12\end{aligned}$$

Solve the second equation for $m\angle WZY$ and use the substitution, $x = 12$.

$$\begin{aligned}m\angle WZY + (8x + 5)^\circ &= 180^\circ \\m\angle WZY &= 180^\circ - (8x + 5)^\circ \\m\angle WZY &= (175 - 8x)^\circ \\m\angle WZY &= 175^\circ - (8(12))^\circ \\m\angle WZY &= 175^\circ - 96^\circ \\m\angle WZY &= 79^\circ\end{aligned}$$

23. Use the Triangle Angle-Sum Theorem to find $m\angle QSR$.

$$\begin{aligned}m\angle QSR + 59^\circ + 71^\circ &= 180^\circ \\m\angle QSR &= 50^\circ\end{aligned}$$

$\angle QSR$ and $\angle QPR$ both intercept the same arc. Therefore, $\angle QSR \cong \angle QPR$ by Corollary 1. Thus, $m\angle QPR = 50^\circ$.

24. They are diameters. The angles of a rectangle are 90° , so they intercept semicircles. The endpoints of a diameter form an arc with measure 180° .
25. Use Case 1 of Theorem 10-12 to write an equation and solve for QR .

$$\begin{aligned}10(QR) &= 5(12) \\QR &= \frac{60}{10} \\QR &= 6\end{aligned}$$

26. Use Theorem 10-10 to write an equation and solve for $m\angle NRP$.

$$\begin{aligned}m\angle NRP &= \frac{1}{2} \left(m\widehat{NP} + m\widehat{QM} \right) \\ &= \frac{1}{2} (81^\circ + 43^\circ) \\ &= \frac{1}{2} (124^\circ) \\ &= 62^\circ\end{aligned}$$

27. Use Case 3 of Theorem 10-12 to write an equation and solve for WZ .

$$\begin{aligned}(WZ)^2 &= WX(WX + XY) \\ (WZ)^2 &= 8(8 + 10) \\ (WZ)^2 &= 8(18) \\ (WZ)^2 &= 144 \\ WZ &= 12\end{aligned}$$

28. Use Case 2 of Theorem 10-11 to write an equation and solve for $m\widehat{XZ}$.

$$\begin{aligned}m\angle YWZ &= \frac{1}{2} \left(m\widehat{YZ} - m\widehat{XZ} \right) \\ 74^\circ &= \frac{1}{2} (187^\circ - m\widehat{XZ}) \\ 148^\circ &= 187^\circ - m\widehat{XZ} \\ m\widehat{XZ} &= 39^\circ\end{aligned}$$

29. no; Sample: As a counterexample, the intercepted arcs could be 260° and 60° , so the measure of the angle formed would be 100° .