11. Hugo did the reflection before the translation.

12. The area of the rectangle is the product of its length and width. Rigid motions preserve lengths, so the product will not change. Therefore, the area of $ABCD$ is equal to the area of $A''B''C''D''$.

13. Answers may vary. Sample:

To get from A to B, the robot must move 3 units to the right and 2 units up. This is the translation $T_{\langle 2,3 \rangle}$.

To get from B to C, the robot must move 4 units to the right. This is the translation $T_{\langle 4,0 \rangle}$.

To get from C to D, the robot must move 1 unit to the right and 3 units up. This is the translation $T_{\langle 1,3 \rangle}$.

Putting these three translation together is: $T_{\langle 1,3 \rangle} \circ T_{\langle 4,0 \rangle} \circ T_{\langle 2,3 \rangle}$.

15. To find $T_{\langle 3,-1 \rangle}$ of a point, add 3 to the x-coordinate and subtract 1 from the y-coordinate.

$A\,(5,0) \to A'\,(5+3, 0-1) = A'\,(8,-1)$
$B\,(-1,2) \to B'\,(-1+3, 2-1) = B'\,(2,1)$
$C\,(6,-3) \to C'\,(6+3, -3-1) = C'\,(9,-4)$

16. To find $T_{\langle -4,0 \rangle}$ of a point, subtract 4 from the x-coordinate.

$D\,(3,3) \to D'\,(3-4, 3) = D'\,(-1,3)$
$E\,(-2,3) \to E'\,(-2-4, 3) = E'\,(-6,3)$
$F\,(0,2) \to F'\,(0-4, 2) = F'\,(-4,2)$

17. To find $T_{\langle -10,-5 \rangle}$ of a point, add 3 to the x-coordinate and subtract 1 from the y-coordinate.

$G\,(0,0) \to G'\,(0-10, 0-5) = G'\,(-10,-5)$
$H\,(3,6) \to H'\,(3-10, 6-5) = H'\,(-7,1)$
$J\,(12,-1) \to J'\,(12-10, -1-5) = J'\,(2,-6)$

18. Compare the location of point A and its image A'.

$A(-1,-1)$ and $A'(-6,-2)$

A is 5 units to the left of and 3 units above its image A'.

So, the rule is $T_{\langle -5,3\rangle}(ABC) = A'B'C'$.

19. Answers may vary. Sample:

$T_{\langle 8,0\rangle} \circ T_{\langle 0,-5\rangle}$

$$\begin{aligned} T_{\langle 8,0\rangle} \circ T_{\langle 0,-5\rangle}(x,\ y) &= T_{\langle 8,0\rangle}(x,\ y-5) \\ &= (x+8,\ y-5) \\ &= T_{\langle 8,-5\rangle}(x,\ y) \end{aligned}$$

20. Note that $x = -2$ and $x = 2$ are parallel lines. This means that the distance between the preimage and image of a point after the translation $R_n \circ R_p(x,y)$ is twice the distance between n and p.

Since n and p are vertical lines, the distance between them is the absolute value of the difference of their x-coordinates.

$$\begin{aligned} d &= |-2-2| \\ &= |-4| \\ &= 4 \end{aligned}$$

So the distance between X and X' is $2d = 2(4) = 8$.

Now we need to determine whether the image is to the left or right of the preimage. $R_{x=2}$ keeps the triangle to the right of the y-axis, but $R_{x=-2}$ places the triangle to the left of the y-axis. This means that X' is 8 units to the left of X. So $R_n \circ R_p(x,y) = T_{\langle 8,0\rangle}$.

21. To write $T_{\langle -3,3\rangle} \circ T_{\langle -2,4\rangle}$ as a single translation, perform both translations on an arbitrary point starting with the rightmost one and simplify the resulting points.

$$\begin{aligned} T_{\langle -3,3\rangle} \circ T_{\langle -2,4\rangle}(x,\ y) &= T_{\langle -3,3\rangle}(x-2,\ y+4) \\ &= (x-2-3,\ y+4+3) \\ &= (x-5,\ y+7) \\ &= T_{\langle -5,7\rangle}(x,\ y) \end{aligned}$$

22. To write $T_{\langle -4,-3\rangle} \circ T_{\langle 3,1\rangle}$ as a single translation, perform both translations on an arbitrary point starting with the rightmost one and simplify the resulting points.

$$\begin{aligned} T_{\langle -4,-3\rangle} \circ T_{\langle 3,1\rangle}(x,\ y) &= T_{\langle -4,-3\rangle}(x+3,\ y+1) \\ &= (x+3-4,\ y+1-3) \\ &= (x-1,\ y+2) \\ &= T_{\langle -1,2\rangle}(x,\ y) \end{aligned}$$

23. To write $T_{\langle 5,-6\rangle} \circ T_{\langle -7,5\rangle}$ as a single translation, perform both translations on an arbitrary point starting with the rightmost one and simplify the resulting points.

$$\begin{aligned} T_{\langle 5,-6\rangle} \circ T_{\langle -7,5\rangle}(x,\ y) &= T_{\langle 5,-6\rangle}(x-7,\ y+5) \\ &= (x-7+5,\ y+5-6) \\ &= (x-2,\ y-1) \\ &= T_{\langle -2,-1\rangle}(x,\ y) \end{aligned}$$

24. To write $T_{\langle 8,-2\rangle} \circ T_{\langle -4,9\rangle}$ as a single translation, perform both translations on an arbitrary point starting with the rightmost one and simplify the resulting points.

$$\begin{aligned} T_{\langle 8,-2\rangle} \circ T_{\langle -4,9\rangle}(x,\ y) &= T_{\langle 8,-2\rangle}(x-4,\ y+9) \\ &= (x-4+8,\ y+9-2) \\ &= (x+4,\ y+7) \\ &= T_{\langle 4,7\rangle}(x,\ y) \end{aligned}$$

25. Given k is the line with equation $x = -3$ and ℓ is the line with equation $x = -2$, solve for $R_k \circ R_1$.

$$R_l(x,\ y) = (-4 - x,\ y)$$

$$R_k(-4 - x,\ y) = (-6 - (-4) - x, y)$$

$$R_k \circ R_l = (-2 - x, y)$$

$$\text{or} \quad T_{\langle -2,0 \rangle}$$

26. Given m is the line with equation $x = 1$ and n is the line with equation $x = -1$, solve for $R_m \circ R_n$.

$$R_n(x,\ y) = (-2 - x,\ y)$$

$$R_m(x,\ y) = (2 - x, y)$$

$$R_m \circ R_n = R_m(-2 - x, y)$$

$$= (2 - (-2) - x, y)$$

$$= (4 - x, y)$$

$$\text{or} \quad T_{\langle 4,0 \rangle}$$

27. Given p is the line with equation $y = 1$ and q is the line with equation $y = 3$, solve for $R_p \circ R_q$.

$$R_p(x,\ y) = (x, 2 - y)$$

$$R_q(x,\ y) = (x, 6 - y)$$

$$R_p \circ R_q = R_p(x,\ 6 - y)$$

$$= (x, 2 - 6 - y)$$

$$= (x, -4 - y)$$

$$\text{or } T_{\langle 0,-4\rangle}$$

28. Given s is the line with equation $y = 2$and t is the line with equation $y = -4$, solve for $R_s \circ R_t$.

$$R_s(x,\ y) = (x, 4 - y)$$

$$R_t(x,\ y) = (x, -8 - y)$$

$$R_s \circ R_t = R_s(x,\ -8 - y)$$

$$= (x, 4 - -6 - y)$$

$$= (x, 12 - y)$$

$$\text{or } T_{\langle 0,12\rangle}$$