18. $m\angle P = 51^\circ$; $m\angle Q = 78$; By Theorem 4-1, we know that $\angle P \cong \angle R$ since we are given $\overline{PQ} \cong \overline{QR}$. Given that $m\angle R = 51$, we can solve for the unknown angle measurements.

$m\angle P = m\angle R$
$m\angle P = 51$
$m\angle Q + m\angle P + m\angle R = 180$
$m\angle Q + 51 + 51 = 180$
$m\angle Q = 78$

19. $m\angle S = 56$; $m\angle U = 56$; By Theorem 4-1, we know that $\angle S \cong \angle U$ since we are given $\overline{ST} \cong \overline{TU}$. Given that $m\angle T = 68$, we can solve for the unknown angle measurements.

$m\angle S + m\angle T + m\angle U = 180$
$m\angle S + 68 + m\angle R = 180$
$m\angle S + m\angle R = 112$
$2(m\angle S) = 112$
$m\angle S = 56$
$m\angle S = m\angle U = 56$

20. $DE = 20$; $EF = 20$; $DF = 29$; By Theorem 4-1, we know that $\overline{DE} \cong \overline{EF}$ since we are given $\angle D \cong \angle F$. Given that $DE = 20$ and $EF = 4x - 12$, solve for the variable x.

$EF = DE$
$4x - 12 = 20$
$4x = 32$
$x = 8$

We then use the value of x to solve for the lengths of the sides of the triangle.

$EF = 4x - 12 = 4(8) - 12 = 32 - 12 = 20$
$FD = 3x + 5 = 3(8) + 5 = 24 + 5 = 29$
$DE = 20$

21. $GH = 45$; $HJ = 45$; $GJ = 29$; By Theorem 4-1, we know that $\overline{GH} \cong \overline{HJ}$ since we are given $\angle G \cong \angle J$. Given that $GH = 45$ and $HJ = 6x + 3$, solve for the variable x.

$$HJ = GH$$
$$6x + 3 = 45$$
$$6x = 42$$
$$x = 7$$

We then use the value of x to solve for the lengths of the sides of the triangle.

$$HJ = 6x + 3 = 6(7) + 3 = 42 + 3 = 45$$
$$JG = 4x + 1 = 4(7) + 1 = 28 + 1 = 29$$
$$GH = 45$$

22. 20; Given $m\angle DFE = 70$, we can apply Theorems 4-1 and 4-2 to solve for $m\angle DEG$.

$$m\angle DEF + m\angle DFE + m\angle FDE = 180$$
$$2m\angle DFE + m\angle DEF = 180$$
$$2\,(70) + m\angle DEF = 180$$
$$m\angle DEF = 40$$
$$2m\angle DEG = 40$$
$$m\angle DEG = 20$$

23. $16\sqrt{2}$; Given $a = 8$ and $c = 24$, we can apply Theorem 4-2 to identify b as a perpendicular bisector of DF and solve for b using the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$b = \sqrt{c^2 - a^2}$$
$$b = \sqrt{(24)^2 - 8^2}$$
$$b = 16\sqrt{2}$$

24. Given : $\overline{AD} \cong \overline{BD} \cong \overline{CD}$

Answers may vary. Sample:

$\triangle ADB$ and $\triangle CDB$ are isosceles by the definition of isosceles triangles.

Let $m\angle DAB = x$.

$m\angle DBA = x$ by the Isosceles Triangle Theorem.

$m\angle CDB = 2x$ by the Triangle Exterior Angle Theorem.

By the Triangle Angle-Sum Theorem,

$$m\angle BCD + m\angle CDB + m\angle DBC = 180$$
$$2 \times m\angle CDB + 2x = 180$$
$$m\angle CDB = \frac{180-2x}{2}$$
$$m\angle CDB = 90 - x.$$

By the Triangle Angle-Sum Theorem,

$$m\angle ABC + m\angle BCA + m\angle CAB = 180$$
$$m\angle ABC + (90 - x) + x = 180$$
$$m\angle ABC + 90 = 180$$
$$m\angle ABC = 90.$$

Therefore, $\triangle ABC$ is a right triangle because $m\angle ABC = 90$.

25. 32; Applying the definition of equilateral triangles, we know that $\angle QPS \cong \angle SQP \cong \angle PSQ$ and each angle has a value of 60. We can then solve for the $m\angle RSQ$.

$$m\angle PSQ + m\angle RSQ = m\angle PSR$$
$$60 + m\angle RSQ = 134$$
$$m\angle RSQ = 74$$

Looking at the graph, we also know that $\overline{SQ} \cong \overline{QR}$ and can apply Theorem 4-1 to solve for $m\angle SQR$.

$$m\angle SQR + m\angle RSQ + m\angle SRQ = 180$$
$$2(m\angle RSQ) + m\angle SRQ = 180$$
$$2(74) + m\angle SRQ = 180$$
$$m\angle SRQ = 32$$

27. a. 5.8 feet; Using the definition of an equilateral triangle, we know that all sides of the triangle are equal. By Theorem 4-1 and 4-2, we know that the triangle is also an isosceles triangle, and therefore the perpendicular bisector of length 5 feet intersects the center of the base of the triangle. The triangle side length, x , can be found as follows.

$$5^2 + \left(\frac{x}{2}\right)^2 = x^2$$

$$x^2 - \frac{x^2}{4} = 25$$

$$\frac{3x^2}{4} = 25$$

$$x^2 = \frac{100}{3}$$

$$x \approx 5.8$$

b. The tent is equilateral and therefore also isosceles. The perpendicular bisector of the bottom of the tent is also the altitude, so a right triangle is formed and the length can be found with the Pythagorean Theorem.