16. Prove Theorem 7-4 using the given right angles, the reflexive property, and the AA~ postulate.

$\triangle ADB$ and $\triangle BDC$; $\angle ADB \cong \angle ABC$ and $\angle DAB \cong \angle BAC$ (or $\angle A \cong \angle A$), so $\triangle ADB \sim \triangle ABC$ by AA~. $\angle BDC \cong \angle ABC$ and $\angle BCD \cong \angle ACB$ (or $\angle C \cong \angle C$), so $\triangle BDC \sim \triangle ABC$ by AA~.

17. 60 and 25; the triangles are congruent by Theorem 7-4 so a proportion of the corresponding legs can be used. Use Corollary 1 to solve for h, and then use Corollary 2 to solve for x.

$$\frac{h}{144} = \frac{65}{156}$$
$$h = 144 \cdot \frac{65}{156}$$
$$h = 60$$

$$\frac{x}{h} = \frac{65}{156}$$
$$\frac{x}{60} = \frac{65}{156}$$
$$x = 60 \cdot \frac{65}{156}$$
$$x = 25$$

18. $6\sqrt{2}$; By Corollary 1 to Theorem 7-4, the length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse. Applying Corollary 1, solve for the length of the altitude y by setting up the following proportions.

$$\frac{6}{y} = \frac{y}{12}$$
$$y^2 = 6 \cdot 12$$
$$y^2 = 72$$
$$y = 6\sqrt{2}$$

19. $a = 16$ and $b = 49$; By Corollary 2 to Theorem 7-4, the altitude to the hypotenuse of a right triangle divides the hypotenuse so that the length of a given leg is the geometric mean of the length of the hypotenuse and the length of the segment of the hypotenuse that is adjacent to the leg. Applying Corollary 2, solve for a.

$$\frac{a}{20} = \frac{20}{25}$$
$$a = \frac{20^2}{25}$$
$$a = 16$$

Use Corollary 2 to Theorem 7-4 to solve for b.

$$\frac{b}{175} = \frac{175}{625}$$
$$b = \frac{175^2}{625}$$
$$b = 49$$

20. **a.** 8; By Corollary 1 to Theorem 7-4, $\frac{m-4}{8} = \frac{8}{2m}$, so $(m-4)(2m) = (8)(8)$ and $2m^2 - 8m - 64 = 0$. Thus, $m = 8$ and $m = -4$. The solution $m = -4$ is extraneous because distance must be positive; therefore $m = 8$.

b. 9; By Corollary 2 to Theorem 7-4, $\frac{9}{n+3} = \frac{n+3}{9+7}$, so $(n+3)^2 = (9)(16)$ and $n^2 + 6m - 135 = 0$ Thus, $n = 9$ and $n = -15$. The solution $n = -15$ is extraneous because distance must be positive, therefore $n = 9$.

21. 8; Answers may vary. Sample: All right triangles in the figure are similar by Theorem 7-4. The leg labeled w in the triangle with hypotenuse length 10 corresponds to the leg labeled 4 in the $3 - 4 - 5$ triangle. Using Corollary 2 to Theorem 7-4, solve for w.

$$\frac{w}{10} = \frac{4}{5}$$
$$w = \frac{40}{5} = 8$$