1. The sides and angles of a triangle have bisectors that are concurrent at specific points. The longest side of a triangle is opposite the largest angle. If two triangles have two congruent sides, the longer third side is opposite the larger included angle.

2. incenter

3. concurrent

4. orthocenter

5. altitude

6. equidistant

7. median

8. By the Converse of the Perpendicular Bisector Theorem, $QR = PQ$ since $RS = PS$.

Set $QR = PQ$ and solve for x.

$$3x + 5 = 6x - 10$$
$$15 = 3x$$
$$x = 5$$

Solve for PR by substituting in $x = 5$.

$$PR = PQ + QR$$
$$= 3(5) + 5 + 6(5) - 10$$
$$= 15 + 5 + 30 - 10$$
$$= 40$$

9. The diagram indicates that $PQ = QR$. By the Perpendicular Bisector Theorem, $RS = PS$. Solve for x.

$$4x + 8 = 5x - 3$$
$$x = 11$$

Substitute $x = 11$ into $PS = 4x + 8$ and then simplify.

$$\begin{aligned} PS &= 4x + 8 \\ &= 4(11) + 8 \\ &= 44 + 8 \\ &= 52 \end{aligned}$$

10. Use the Pythagorean theorem to find JK and KL.

$$12^2 + JK^2 = KM^2$$
$$JK^2 = KM^2 - 144$$
$$JK = \sqrt{KM^2 - 144}$$

$$12^2 + KL^2 = KM^2$$
$$KL^2 = KM^2 - 144$$
$$KL = \sqrt{KM^2 - 144}$$

Therefore $JK = KL$.

By the Converse of the Angle Bisector Theorem, $m\angle JMK = m\angle KML$.

Therefore $m\angle KML = 25^\circ$.

11. The diagram indicates that $JK = 17$. We know that $m\angle JML = m\angle JMK + m\angle KML$ and we are given that $m\angle JMK = 24.5^\circ$. Solve for $m\angle KML$.

$$m\angle JML = m\angle JMK + m\angle KML$$
$$49^\circ = 24.5^\circ + m\angle KML$$
$$m\angle KML = 24.5^\circ$$

Therefore, $m\angle KML = m\angle JMK$.

By the Angle Bisector Theorem, $JK = KL$. Therefore, $KL = 17$.

12. Let x be half the length of the segment. Use the Pythagorean Theorem to solve for x.

$$x^2 + 5^2 = 7^2$$
$$x^2 = 24$$
$$x = \sqrt{24}$$
$$x = 4.9$$

The length of entire segment is $2(4.9) = 9.8$ cm.

13. The incenter is the point where the angle bisectors of a triangle intersect. Point K is an incenter.

14. The circumcenter is the point where the perpendicular bisectors of a triangle intersect. Point A is a circumcenter.

15. Use the Triangle Angle-Sum Theorem to find $m\angle FGE$.

$$m\angle FEG + m\angle EFG + m\angle FGE = 180^\circ$$
$$36^\circ + 36^\circ + 34^\circ + 34^\circ + m\angle FGE = 180^\circ$$
$$m\angle FGE = 40^\circ$$

By Theorem 5-6, $\overline{JG}$ is an angle bisector of $\angle FGE$, and therefore,

$$m\angle FGL = \frac{1}{2} m\angle FGE$$
$$= \frac{1}{2}(40^\circ)$$
$$= 20^\circ$$

16. The diagram indicates that $JL = 5$. We know that $\overline{EL}$, $\overline{FL}$, and $\overline{LG}$ are angle bisectors of $\triangle EFG$. By the Concurrency of Angle Bisectors Theorem, $JL = KL$. Therefor $KL = 5$.

17. If the circumcenter is on a side of the triangle, the triangle is a right triangle, and the circumcenter must be at the midpoint of the hypotenuse of the right triangle. The area of the circle is πr^2, where r is half the length of the hypotenuse.

18. A median of a triangle is a line segment from the midpoint of one side to the opposite vertex. W is the midpoint of $\overline{DE}$. $\overline{WF}$ extends from the midpoint of $\overline{DE}$ to the opposite vertex F, so it is a median.

19. An altitude of a triangle is a line segment perpendicular to one side and ending at the opposite vertex. $\overline{EX}$ is perpendicular to $\overline{DF}$ and ends at the opposite vertex E, so it is an altitude.

20. The points $(2, 0)$ and $(2, 12)$ have the same x -value, so the line connecting them is vertical.

The altitude perpendicular to this vertical line that passes through $(8, 6)$ is the horizontal line with equation $y = 6$.

The slope of the line connecting $(2, 12)$ and $(8, 6)$ is

$$\frac{6-12}{8-2} = -\frac{6}{6} = -1.$$

The slope of the line perpendicular to this one has slope 1.

The altitude perpendicular to this line that passes through $(2, 0)$ has equation:

$$y - 0 = 1(x - 2)$$
$$y = x - 2$$

Solve the system of equations to find the intersection point.

$$y = 6$$
$$y = x - 2$$

Set the equations equal and then solve for x.

$$6 = x - 2$$
$$x = 8$$

Use the value of x to solve for y.

$$y = x - 2$$
$$= 8 - 2$$
$$= 6$$

The orthocenter is at $(8, 6)$.

21. Find the slope of the line connecting the points $(7, 8)$ and $(9, 6)$.

$$\frac{6-8}{9-7} = -\frac{2}{2} = -1$$

The slope of the line that is perpendicular to the line is 1.

The altitude perpendicular to the line that passes through $(5, 4)$ has equation:

$$y - 4 = 1(x - 5)$$
$$y = x - 5 + 4$$
$$y = x - 1$$

Find the slope of the line connecting the points $(7, 8)$ and $(5, 4)$.

$$\frac{4-8}{5-7} = \frac{-4}{-2} = 2$$

The slope of the line that is perpendicular to this one is $-\frac{1}{2}$.

The altitude perpendicular to this line that passes through $(9, 6)$ has equation:

$$y - 6 = -\frac{1}{2}(x - 9)$$
$$y = -\frac{1}{2}x + \frac{9}{2} + 6$$
$$y = -\frac{1}{2}x + \frac{21}{2}$$

Solve the system of equations to find the intersection point.

$$y = x - 1$$
$$y = -\frac{1}{2}x + \frac{21}{2}$$

Set the equations equal and then solve for x.

$$-\frac{1}{2}x + \frac{21}{2} = x - 1$$
$$\frac{23}{2} = \frac{3}{2}x$$
$$x = \frac{23}{3}$$

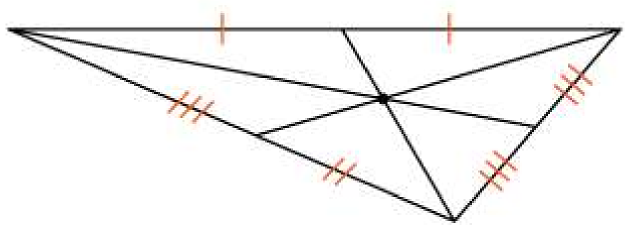
Now substitute this value into $y = x - 1$ and simplify.

$$y = \frac{23}{3} - 1$$
$$y = \frac{20}{3}$$

The orthocenter is at $\left(\frac{23}{3}, \frac{20}{3}\right)$.

22. If the orthocenter is on a vertex, then $\triangle ABC$ must be a right triangle. So $m\angle ABC = 90^\circ$.

23. The center of gravity of a triangle is at the centroid. The centroid is the point where the medians intersect. The medians extend from the midpoint of each side to the opposite vertex.



24. Use the Triangle Inequality Theorem to check if the sum of any two lengths is greater than the third length.

$$14 + 32 = 46 > 18$$
$$18 + 32 = 50 > 13$$
$$18 + 14 = 32$$

Since $18 + 14 \leq 32$, these lengths can't form a triangle.

25. Use the Triangle Inequality Theorem to check if the sum of any two lengths is greater than the third length.

$$14 + 25 = 39 > 29$$
$$14 + 29 = 43 > 25$$
$$25 + 29 = 54 > 14$$

The sum of any two lengths is greater than the third length, so these lengths can form a triangle.

26. Use the Triangle Inequality Theorem to check if the sum of any two lengths is greater than the third length.

$$37 + 22 = 59 > 56$$
$$37 + 56 = 93 > 22$$
$$22 + 56 = 78 > 37$$

The sum of any two lengths is greater than the third length, so these lengths can form a triangle.

27. Use the Triangle Inequality Theorem to check if the sum of any two lengths is greater than the third length.

$$87 + 35 = 122 > 41$$
$$35 + 41 = 76 < 87$$
$$41 + 87 = 129 > 35$$

$35 + 41$ is not greater than 87, so these lengths can't form a triangle.

28. According to Theorem 5-9, the angle with the least measure lies opposite the shortest side. By looking at the figure, we can see that the shortest side is $\overline{PR}$. The angle opposite $\overline{PR}$ is $\angle Q$, so it is the smallest angle.

29. According to Theorem 5-9, the angle with the greatest measure lies opposite the longest side. By looking at the figure, we can see that the longest side is $\overline{PQ}$. The angle opposite $\overline{PQ}$ is $\angle R$, so it is the largest angle.

30. If the third side were smaller than the sum of the other two sides, the triangle would not close. If it were equal to the sum of the other two sides, the two sides would form the third side.

31. Let x be the third side of the garden. Apply the Triangle Inequality Theorem.

$$6.4 + 8.2 > x$$
$$6.4 + x > 8.2$$
$$8.2 + x > 6.4$$

If $6.4 + 8.2 > x$, then $x < 14.6$.

If $6.4 + x > 8.2$, then $x > 1.8$.

$8.2 + x > 6.4$ is true for any positive value of x. So x is greater than 0.

Therefore, $1.8 < x < 14.6$.

The length of the third side is between 1.8 m and 14.8 m.

The cost of the third side is between $1.8 \times 29.25 = \$52.65$ and $14.6 \times 29.25 = \$427.05$.

32. By the Hinge Theorem, $3x - 1 < 23$. Solve for x.

$$3x - 1 < 23$$
$$3x < 24$$
$$x < 8$$

33. By the Hinge Theorem, $4x - 7 > 3x + 6$. Solve for x.

$$4x - 7 > 3x + 6$$
$$x > 13$$

Because the sum of angles in a triangle is 180°:

$(3x + 6) + 63 < 180$		$4x - 7 < 180$
$3x < 111$	and	$4x < 187$
$x < 37$		$x < 46.75$

So $13 < x < 37$.

34. Cameron cannot apply the Converse of the Hinge Theorem because the diagram does not show that the triangles have two pairs of corresponding congruent sides.