8. Yes. The transformation appears to be a rigid motion because the side lengths and angles of the preimage appear to have been preserved in the image.

9. The rule for reflecting a point across the *y*-axis is $r_{y-axis}(x, y) = (-x, y)$. Apply this rule to points $H(-3, 2)$, $J(-1, -3)$ and $K(4, 3)$.

$r_{y-axis}(-3, 2) = (3, 2)$
$r_{y-axis}(-1, -3) = (1, -3)$
$r_{y-axis}(4, 3) = (-4, 3)$

The coordinates of the reflected points are $H'(3, 2)$, *J*'(1,-3), and *K*'(-4,3).

10. The rule for reflecting a point across the *y*-axis is $r_{x-axis}(x, y) = (x, -y)$. Apply this rule to points $H(-3, 2)$, $J(-1, -3)$ and $K(4, 3)$.

$r_{x-axis}(-3, 2) = (-3, -2)$
$r_{x-axis}(-1, -3) = (-1, 3)$
$r_{x-axis}(4, 3) = (4, -3)$

The coordinates of the reflected points are $H'(-3, -2)$, *J*'(-1,3), and *K*'(4,-3).

11. The slope of the line through the two points should be the negative reciprocal of the slope of the line, and the midpoint of the two points is a point on the line.

12. $T_{<-3,2>}$ translates a point to the left 3 units and up 2 units. This can be written as: $T_{<-3,2>}(x, y) = (x - 3, y + 2)$.

Apply this formula to the points given.

$T_{<-3,2>}(-4, 3) = (-4 - 3, 3 + 2)$
$= (-7, 5)$
$T_{<-3,2>}(-2, 3) = (-2 - 3, 3 + 2)$
$= (-5, 5)$
$T_{<-3,2>}(1, -3) = (1 - 3, -3 + 2)$
$= (-2, -1)$

The coordinates of the translated points are $P'(-7, 5)$, *Q*'(-5,5), and *R*'(-2,-1).

13. $T_{<4,-5>}$ translates a point to the right 4 units and down 5 units. This can be written as: $T_{<4,-5>}(x, y) = (x + 4, y - 5)$.

Apply this formula to the points given.

$T_{<4,-5>}(-4, 3) = (-4 + 4, 3 - 5)$
$= (0, -2)$
$T_{<4,-5>}(-2, 3) = (-2 + 4, 3 - 5)$
$= (2, -2)$
$T_{<4,-5>}(1, -3) = (1 + 4, -3 - -5)$
$= (5, -8)$

The coordinates of the translated points are $P'(0, -2)$, Q'(2,-2), and R'(5,-8).

14. We have points S(0, 1), T(2, 1), U(4, –2), V(2, –3) and S'(–4, 2), T'(–3, 3), U'(–1, 0), $V'(-3, -1)$.

Subtract the x value of T from the x value of T' to find the horizontal translation.

–3 – 2 = – 5

Subtract the y value of T from the y value of T′ to find the vertical translation.

3 – 1 = 2

There is a horizontal translation of – 5 and a vertical translation of 2. This is written as $T_{<-5,2>}$.

15. The distance of the translation will be twice the difference between the lines x = 2 and x =–3.

2(2 – –3) = 2(5) = 10

The distance is 10 units.

16. The formula for rotating points 90° about the origin is $r_{(90^\circ,O)}(x,y) \rightarrow (-y,x)$.

Apply this formula to the given points.

$$r_{(90^\circ,O)}(2,-2) = (2,2)$$
$$r_{(90^\circ,O)}(-3,-2) = (2,-3)$$
$$r_{(90^\circ,O)}(-1,3) = (-3,-1)$$

This gives the points $A'(2, 2)$, $B'(2, -3)$, and $C'(-3, -1)$.

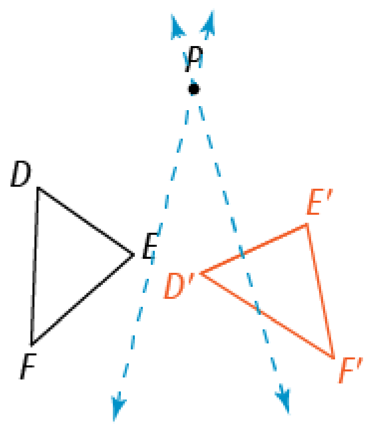
17. The formula for rotating points 270° about the origin is $r_{(270^\circ,O)}(x,y) \rightarrow (y,-x)$.

Apply this formula to the given points.

$$r_{(270^\circ,O)}(2,-2) = (-2,-2)$$
$$r_{(270^\circ,O)}(-3,-2) = (-2,3)$$
$$r_{(270^\circ,O)}(-1,3) = (3,1)$$

This gives the points $A'(-2,-2)$, $B'(-2,3)$, and $C'(3,1)$.

18.



19. Since the lines intersect at a 90° angle, a reflection over one of the lines is equivalent to a 90° rotation. The composition of reflections over both lines is equivalent to $2 \times 90^\circ = 180^\circ$. This can be written as $r_{(180^\circ,P)}$.

20. A composition is evaluated from right to left. First, find R_j.

A formula to reflect a point over j is $R_j(x, y) = (6 - x, y)$.

Apply this formula to the points $L(-2, 4)$, $M(1, 2)$, and $N(-3, -5)$.

$R_j(-2, 4) = (8, 4)$
$R_j(1, 2) = (5, 2)$
$R_j(-3, -5) = (9, -5)$

$T_{<-2,4>}$ shifts a point to the left 2 units and up 4 units. A formula for this translation is $T_{<-2,4>}(x, y) = (x - 2, y + 4)$.

Apply this formula to the reflected points.

$T_{<-2,4>}(8, 4) = (6, 8)$
$T_{<-2,4>}(5, 2) = (3, 6)$
$T_{<-2,4>}(9, -5) = (7, -1)$

The coordinates of the reflected points are $L'(6, 8)$, $M'(3, 6)$, and $N'(7, -1)$.

21. A composition is evaluated from right to left. First, find R_k.

A formula to reflect a point over k is $R_k(x, y) = (x, -4 - y)$.

Apply this formula to the points $L(-3, 4)$, $M(1, 2)$, and $N(-3, -5)$.

$R_k(-2, 4) = (-2, -8)$
$R_k(1, 2) = (1, -6)$
$R_k(-3, -5) = (-3, 1)$

$T_{<2,-3>}$ shifts a point to the right 2 units and down 3 units. A formula for this translation is $T_{<2,-3>}(x, y) = (x + 2, y - 3)$.

Apply this formula to the reflected points.

$T_{<2,-3>}(-2, -8) = (0, -11)$
$T_{<2,-3>}(1, -6) = (3, -9)$
$T_{<2,-3>}(-3, 1) = (-1, -2)$

The coordinates of the reflected points are $L'(0, -11)$, $M'(3, -9)$, and $N'(-1, -2)$.

22. Look at the graph to find the points $Q(2, -1)$, $R(4, -2)$, and $S(4, -4)$, and $Q'(-1, 4)$, $R'(-3, 3)$, and $S'(-3, 1)$.

Find the midpoints of $\overline{QQ'}$ and $\overline{RR'}$ to find the line of reflection.

The midpoint of $\overline{QQ'}$ is $\left(\frac{2+-1}{2}, \frac{-1+4}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$.

The midpoint of $\overline{RR'}$ is $\left(\frac{4+-3}{2}, \frac{-2+3}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$.

Since both midpoints have the same x-coordinate, the line of reflection is $x = \frac{1}{2}$.

Reflect the points over $x = \frac{1}{2}$ using $R_{x=\frac{1}{2}}(x, y) = (1 - x, y)$.

$R_{x=\frac{1}{2}}(2, -1) = (-1, -1)$
$R_{x=\frac{1}{2}}(4, -2) = (-3, -2)$
$R_{x=\frac{1}{2}}(4, -4) = (-3, -4)$

These points have the same x coordinates as Q', R', and S', but the y coordinates are different. This shows that we will need a vertical shift to complete the glide reflection.

To find the vertical shift, find the difference between the y coordinates of R and R'.

$3 - (-2) = 5$

The points must be shifted upwards 5 units. This can be written as $T_{<0,5>}$.

The entire glide reflection is $R_m \circ T_{<0,5>}$ where m is the line $x = \frac{1}{2}$.

23. Yes. The translation and reflection can be performed in either order, so a glide reflection can be described as a composition of a translation and then a reflection.

24. A horizontal line that passes through the center of the circle would divide the figure into equal halves. A vertical line that passes through the center of the circle would also divide the figure into equal halves. A reflection across either of these lines would map the figure onto itself.

A 180° rotation about the center of the circle would map the figure onto itself because the blue shape on the right would be rotated so that it is in the position of the blue shape on the left.

25. This figure does not have any reflectional symmetry. Any line drawn would not divide the figure into equal halves. A 180° rotation about the center of the figure would map the figure onto itself, so it has rotational symmetry, specifically point symmetry.

26. The figure has reflectional symmetry with lines of reflection that go through the far vertex of each triangle and the center of the figure. The figure has rotational symmetry about the center of the figure of 120° and 240°.

27. Since the figure has two lines of symmetry, it must have rotational symmetry because the composition of the reflections across two lines of symmetry is a rotation about the center of the figure that will map the figure onto itself.