1. Answers may vary. Sample: The fundamentals of geometry include the undefined and defined terms such as a point, line, plane, ray, segment, and angle. They include measuring segments and angles, as well being able to construct certain geometric figures. Other fundamentals are using direct and indirect reasoning to prove conjectures in geometry.

2. A statement accepted without proof is a **postulate.**

3. Arriving at a conclusion by observing patterns is **inductive reasoning.**

4. A **biconditional** is the combination of the conditional and its converse.

5. According to the **Law of Detachment,** if a conditional statement and its hypothesis are true, then its conclusion is also true.

6. A conjecture that has been proven is a **theorem.**

7. A statement of the form *if not q, then not* p is a **contrapositive** of the conditional *if p, then q.*

8. You use **deductive reasoning** when you logically come to a valid conclusion based on given statements.

9. Use the Segment Addition Postulate and the given segment lengths to write and solve an equation to find $x$.

   $LM + MN = LN$
   $(3x + 2) + (x + 19) = 45$
   $4x + 21 = 45$
   $4x = 24$
   $x = 6$

10. First, solve for $x$. Then, use the value of $x$ to solve for $QS$.

   $12x - 21 = RS$
   $12x - 21 = 27$
   $12x = 48$
   $x = 4$

   $QS = QR + RS$
   $QS = 9x + 27$
   $QS = 63$

**11.** Use the Angle Addition Postulate and the given information to write and solve an equation to find $m\angle EBF$.

$$m\angle EBF + m\angle FBG = m\angle EBG$$
$$m\angle EBF + 2(m\angle EBF) = 60^{\circ}$$
$$3(m\angle EBF) = 60^{\circ}$$
$$m\angle EBF = 20^{\circ}$$

**12.** Use the Angle Addition Postulate and the given information to write and solve an equation to find $m\angle DBE$.

$$m\angle ABD + m\angle DBE = m\angle ABE$$
$$m\angle DBE + m\angle DBE = 64^{\circ}$$
$$m\angle DBE = 32^{\circ}$$

**13.** Use the Angle Addition Postulate and the given information to write and solve an equation to find $m\angle GBC$.

$$m\angle GBC = m\angle GBH + m\angle HBC$$
$$m\angle GBC = m\angle GBH + m\angle GBH$$
$$m\angle GBC = 28^{\circ} + 28^{\circ}$$
$$m\angle GBC = 56^{\circ}$$

**14.** Use the Segment Addition Postulate and the given information to write and solve equations to find the coordinate of $J$.

Start by finding $KL$.

$$KL = |K - L|$$
$$= |7 - 23|$$
$$= 16$$

Now, use the fact that $JK = KL$ and the coordinate of $K$ to find the coordinate of $J$.

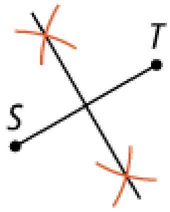
$$KL = L - K = 23 - 7 = 16$$

$$JK = |J - K|$$
$$KL = |J - K|$$
$$16 = |J - 7|$$

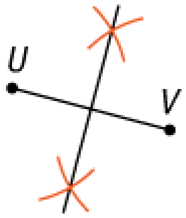
$$\begin{array}{lcl} 16 = J - 7 & & 16 = -(J-7) \\ 23 = J & \text{or} & 16 = 7 - J \\ & & J = -9 \end{array}$$

Since $JL$ exists, $J \neq 23$. Therefore, $J = -9$.

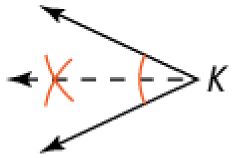
**15.**



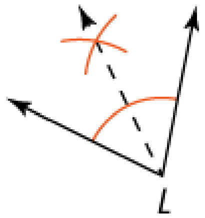

**16.**




**17.**

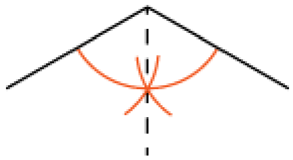



**18.**




**19.** The arcs will not intersect if the compass width is less than half of the segment width.

**20.**



**21.** Let $(x_1, y_1) = E(-3, 2)$ and $(x_2, y_2) = F(1, 3)$.

$$\text{Midpoint of } \overline{EF} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{-3+1}{2}, \frac{2+3}{2}\right)$$
$$= \left(-1, \frac{3}{2}\right)$$

$$EF = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(1 - (-3))^2 + (3 - 2)^2}$$
$$= \sqrt{4^2 + 1^2}$$
$$= \sqrt{17}$$

**22.** Let $(x_1, y_1) = G(5,1)$ and $(x_2, y_2) = F(1,3)$.

Midpoint of $\overline{FG} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
$= \left(\frac{5+1}{2}, \frac{1+3}{2}\right)$
$= (3,2)$

$FG = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(1-5)^2 + (3-1)^2}$
$= \sqrt{(-4)^2 + 2^2}$
$= \sqrt{20}$
$= 2\sqrt{5}$

**23.** Let $(x_1, y_1) = G(5,1)$ and $(x_2, y_2) = H(1,-1)$.

Midpoint of $\overline{GH} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
$= \left(\frac{5+1}{2}, \frac{1+(-1)}{2}\right)$
$= (3,0)$

$GH = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(1-5)^2 + (-1-1)^2}$
$= \sqrt{(-4)^2 + (-2)^2}$
$= \sqrt{20}$
$= 2\sqrt{5}$

**24.** Let $(x_1, y_1) = E(-3,2)$ and $(x_2, y_2) = H(1,-1)$.

Midpoint of $\overline{EH} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
$= \left(\frac{-3+1}{2}, \frac{2+(-1)}{2}\right)$
$= \left(-1, \frac{1}{2}\right)$

$EH = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(1-(-3))^2 + (-1-2)^2}$
$= \sqrt{(-4)^2 + (-3)^2}$
$= \sqrt{25}$
$= 5$

**25.** Horizontal distance

$$\frac{2}{5}(x_2 - x_1) = \frac{2}{5}(-3 - 1)$$
$$= -\frac{8}{5}$$

Vertical Distance

$$\frac{2}{5}(y_2 - y_1) = \frac{2}{5}(2 - (-1))$$
$$= \frac{6}{5}$$

Point

$$K = \left(1 + \left(-\frac{8}{5}\right), -1 + \left(\frac{6}{5}\right)\right)$$
$$= \left(-\frac{3}{5}, \frac{1}{5}\right)$$

**26.** Let the school be the first point and the store be the second point.

$$(x_1, y_1) = (8, 12)$$

$$(x_2, y_2) = H(1, -1)$$

$$\mathrm{M} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{8+14}{2}, \frac{12+3}{2}\right)$$
$$= \left(11, 7\frac{1}{2}\right)$$

**27.** When looking for a pattern, observe that the terms increase and the second term is twice the value of the first term. Also notice that the third term is three times the value of the second term and so on and so forth. Using this pattern, we can deduce:

$6th$ term: $5th$ term $\times\ 6 = 120 \times 6 = 720$
$7th$ term: $6th$ term $\times\ 7 = 720 \times 7 = 5,040$

**28.** When looking for a pattern, observe that the terms increase and the second term is two more than the first term. Also notice that the third term is three more than the second term and that the fourth term is two more than the third term. Thus, we can surmise that the terms are calculated by addition of two and three every other term. Using this pattern, we can deduce:

$6th$ term: $5th$ term $+\ 2 = 13 + 2 = 15$
$7th$ term: $6th$ term $+\ 3 = 15 + 3 = 18$

**29.** When looking for a pattern, observe that the terms increase and the difference of the first two terms is three, the difference of the second and third terms is four, and the difference of the third and fourth terms is five. Using this pattern, we can deduce:

$6th$ term: $5th$ term $+ 7 = 20 + 7 = 27$
$7th$ term: $6th$ term $+ 8 = 27 + 8 = 35$

**30.** When looking for a pattern, observe that the terms increase and the difference of the first two terms is four. Test whether the pattern continues with subsequent terms and if the rule works. Using this pattern, we can deduce:

$6th$ term: $5th$ term $+ 4 = 33 + 4 = 37$
$7th$ term: $6th$ term $+ 4 = 37 + 4 = 41$

**31.** The statement says that all triangles have three congruent angles, and, thus, all three angles have the same measure. One counterexample is a right triangle because a right triangle has one angle that measures $90^\circ$ and two supplementary angles.

**32.** Answers may vary. Sample: 2 and 14; 4 and 16; 6 and 18; 8 and 20

**33.** A true statement must always be true, so one counterexample shows it is not true. An example only shows that the statement is true in at least one case, but there may be other cases where it is not true.

**34.** To solve the problem, first find how much Jack saves per week. Observe that the amount of money in Jacks bank account increases each week and that the difference between the first two weeks is \$1.75. Testing whether this pattern continues over subsequent weeks, we find that Jack saves \$1.75 per week. So, on week $n$, Jack will have saved \$1.75$n$.

$$
\begin{aligned}
&\text{Week } 10 = 1.75n \\
&= 1.75\,(10) \\
&= 17.5
\end{aligned}
$$

So on week 10, Jack has saved $\$17.50$.

**35.** **Conditional:** If a number is a multiple of $4$, then it is a multiple of $2$.

**Converse:** If a number is a multiple of $2$, then it is a multiple of $4$.

**Inverse:** If a number is not a multiple of $4$, then it is not a multiple of $2$.

**Contrapositive:** If a number is not a multiple of $2$, then it is not a multiple of $4$.

**36.** **Conditional:** If it is Saturday, then Kona jogs $5$ miles.

**Converse:** If Kona jogs $5$ miles, then it is Saturday.

**Inverse:** If it is not Saturday, then Kona does not jog $5$ miles.

**Contrapositive:** If Kona does not jog $5$ miles, then it is not Saturday.

**37.** False; counterexample: $1$ is less than $4$, but it is not prime.

**38.** Start by simplifying the inequality.

$$3x - 7 < 14$$
$$3x < 21$$
$$x < 7$$

True; simplifying the inequality gives $x < 7$, and any number less than 7 is also less than $8$.

**39.** False; counterexample: a day with such little snowfall that school is not cancelled

**40.** Pudding is available if and only if it is a Tuesday.

**41.** We know that if $AB = BC$, then $DE = 2\,(AB)$. We are given $AB = 6$ and $BC = 6$. From the given information, we see that $AB = BC$ is true, so $DE = 2\,(AB)$ is true. If we substitute $AB = 6$, we can conclude that $DE = 12$.

$$DE = 2\,(AB)$$
$$= 2\,(6)$$
$$= 12$$

**42.** The Law of Syllogism states that if $p \to q$ and $q \to r$ are true, then $p \to r$ is true.

Write each of the given statements as conditionals and then find the hypothesis and conclusion of each one.

If it is a sunny day, then the water park is filled with people.

$p$: it is a sunny day

$q$: the water park is filled with people

If the water park is filled with people, then the lines for each ride are long.

Note that the hypothesis of this statement is the conclusion of the previous statement.

$q$: the water park is filled with people

$r$: the lines for each ride are long

Since both conditionals are true, we can then apply the Law of Syllogism and write a true conditional, $p \to r$.

If it is a sunny day, the lines for each ride are long.

**43.** Find the hypothesis and conclusion of the given conditional.

If you use the advertised toothpaste for more than a week, then you will have fresher breath.

$p$: use the advertised toothpaste for more than a week

$q$: have fresher breath

Next use the given information to determine if the hypothesis is true. Since a week is seven days and $10 > 7$, we know the hypothesis is true. Since we know the conditional and the hypothesis are true, we can use the Law of Detatchment to conclude that that the conclusion is true.

Therefore, you can conclude that you have fresher breath.

**44.** Use the Vertical Angles Theorem to set up and solve an equation to find $x$.

$$3x - 6 = 2x + 22$$
$$x = 28$$

Substitute for $x$ to find the angles.

| | |
|---|---|
| $3x - 6 = 3\,(28) - 6$ | $2x + 22 = 2\,(28) + 22$ |
| $= 84 - 6$ | $= 56 + 22$ |
| $= 78$ | $= 78$ |

$x = 28;\ 78^\circ;\ 78^\circ$

**45.** Use the Vertical Angles Theorem to set up and solve an equation to find $x$.

$$3x - 5 = 2x + 32$$
$$x = 37$$

Substitute for $x$ to find the angles.

| | |
|---|---|
| $3x - 5 = 3\,(37) - 5$ | $2x + 32 = 2\,(37) + 32$ |
| $= 111 - 5$ | $= 74 + 32$ |
| $= 106$ | $= 106$ |

$x = 37;\ 106^\circ;\ 106^\circ$

**46.**

| | Statements | Reasons |
|---|---|---|
| 1) | $m\angle TUV = 90^\circ$ | 1) Given |
| 2) | $m\angle TUW + m\angle WUV = m\angle TUV$ | 2) Angle Addition Postulate |
| 3) | $y^\circ + 42^\circ = 90^\circ$ | 3) Substitution Property of Equality |
| 4) | $y^\circ = 48^\circ$ | 4) Subtraction Property of Equality |
| 5) | $4x = y$ | 5) Vertical Angles Theorem |
| 6) | $4x = 48$ | 6) Substitution Property of Equality |
| 7) | $x = 12$ | 7) Simplify. |

**47.** Contrapositive: If $x = 12,$ then $GJ \neq 48$.

Assume $x = 12$. By the Segment Addition Postulate, $GJ = GH + HJ$. Substitute the expressions for the lengths of each segment into the equation to find $GJ$.

$$\begin{aligned} GJ &= 2x + x \\ &= 3x \end{aligned}$$

Substitute for $x$.

$$\begin{aligned} GJ &= 3(12) \\ &= 36 \end{aligned}$$

Since $36 \neq 48$, $GJ \neq 48$ which proves that the contrapositive is true. So, the original conditional must be true.