15. a. $\overline{BD}$ extends from the side opposite B to B, bisecting the angle. This makes it an angle bisector.

b. $\overline{FJ}$ extends from the midpoint of $\overline{HG}$ to the opposite vertex. This makes it a median.

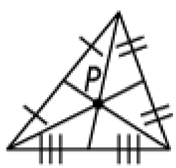
c. $\overline{CE}$ extends from C to the opposite side, perpendicular to $\overline{AB}$. This makes it an altitude.

d. $\overline{KL}$ extends from the midpoint of $\overline{FG}$ to the opposite side, perpendicular to $\overline{FG}$. This makes it a perpendicular bisector.

16. By the Concurrency of Medians Theorem, $LP = \frac{2}{3}KL$. Solve for KL using the substitution $LP = 15$.

$$\frac{2}{3}KL = LP$$
$$KL = \frac{3}{2}LP$$
$$KL = \frac{3}{2}(15)$$
$$KL = \frac{45}{2} = 22.5$$

17. Draw medians from the midpoint of each side to the opposite vertex. The point where the medians intersect is the centroid.



18. a. The altitudes extending from the vertices opposite the legs both pass through the right angle. This means the orthocenter is on the right angle.

b. This is an acute triangle, so the altitudes will meet at a point inside the triangle.

c. This is an acute triangle, so the altitudes will meet at a point inside the triangle.

d. This is an obtuse triangle, so the altitudes will meet at a point outside the triangle.

19. a. Label the points $A(0, 0)$, $B(8, 4)$, and $C(4, 22)$.

Find the slope of $\overline{AB}$.

$\frac{4-0}{8-0} = \frac{1}{2}$

Find the slope of $\overline{AC}$.

$\frac{22-0}{4-0} = \frac{11}{2}$

The slope of the line perpendicular to $\overline{AB}$ is -2. The slope of the line perpendicular to $\overline{AC}$ is $-\frac{2}{11}$.

The equation of the line perpendicular to $\overline{AB}$ that passes through C is

$$y - 22 = -2(x - 4)$$
$$y = -2x + 8 + 22$$
$$y = -2x + 30$$

The equation of the line perpendicular to $\overline{AC}$ that passes through B is

$$y - 4 = -\frac{2}{11}(x - 8)$$
$$y = -\frac{2}{11}x + \frac{16}{11} + 4$$
$$y = -\frac{2}{11}x + \frac{60}{11}$$

Solve the system of equations to find the intersection point.

$$y = -2x + 30$$
$$y = -\frac{2}{11}x + \frac{60}{11}$$
$$-2x + 30 = -\frac{2}{11}x + \frac{60}{11}$$
$$-2x + \frac{2}{11}x = \frac{60}{11} - 30$$
$$-\frac{20}{11}x = -\frac{270}{11}$$
$$x = \frac{270}{20}$$
$$x = 13.5$$

$$y = -2(13.5) + 30$$
$$y = -27 + 30$$
$$y = 3$$

The orthocenter is at $(13.5, 3)$.

b. Label the points $A(3, 1)$, $B(10, 8)$, and $C(5, 13)$.

Find the slope of $\overline{AB}$.

$\frac{1-8}{3-10} = 1$

Find the slope of $\overline{AC}$.

$\frac{1-13}{3-5} = 6$

The slope of the line perpendicular to $\overline{AB}$ is -1. The slope of the line perpendicular to $\overline{AC}$ is $-\frac{1}{6}$.

The equation of the line perpendicular to $\overline{AB}$ that passes through C is

$$\begin{aligned} y - 13 &= -1(x - 5) \\ y &= -x + 5 + 13 \\ y &= -x + 18 \end{aligned}.$$

The equation of the line perpendicular to $\overline{AC}$ that passes through B is

$$\begin{aligned} y - 8 &= -\frac{1}{6}(x - 10) \\ y &= -\frac{1}{6}x + \frac{10}{6} + 8 \\ y &= -\frac{1}{6}x + \frac{29}{3} \end{aligned}.$$

Solve the system of equations to find the intersection point.

$$\begin{aligned} &y = -x + 18 \\ &y = -\frac{1}{6}x + \frac{29}{3} \\ &-\frac{1}{6}x + \frac{29}{3} = -x + 18 \\ &\frac{5}{6}x = 18 - \frac{29}{3} \\ &\frac{5}{6}x = \frac{25}{3} \\ &x = 10 \end{aligned}$$

$$\begin{aligned} y &= -10 + 18 \\ y &= 8 \end{aligned}$$

The orthocenter is at $(10, 8)$.