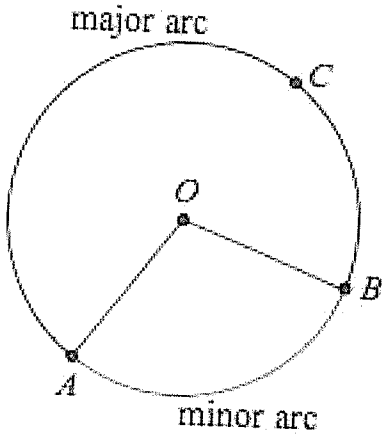
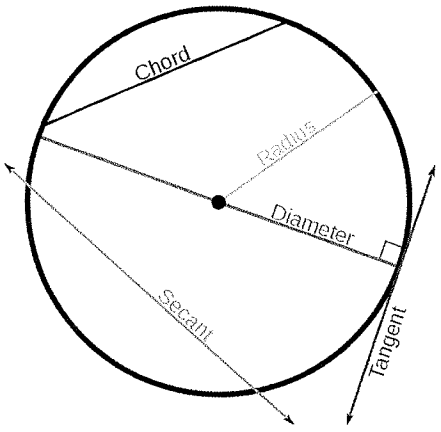
Arcs and Chords

An **arc** is a part of a circle. If an arc is above the central angle and if the central angle is less than 180°, then we call this a **minor arc**. If an arc is above the central angle and greater than 180°, we call this a **major arc**. In the diagram on the right, arc AB is the minor arc, and arc ACB is the major arc.



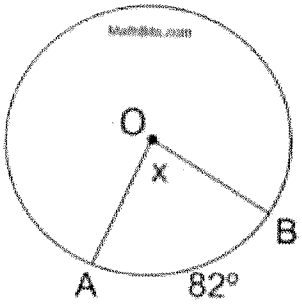
A **chord** is a straight line segment whose endpoints both lie on the circle. A **secant line** (or just secant) is the infinite line extension of a chord. A **tangent** is similar to a secant in that it is an infinite line, but it only intersects the circle at one point. In the diagram on the right, we can see the radius and diameter of the circle, as well as a secant, chord, and tangent.



Formulas for Circle Angles

1) Central Angle

A central angle is an angle formed by two radii with the vertex at the center of the circle



Central Angle = Intercepted Arc

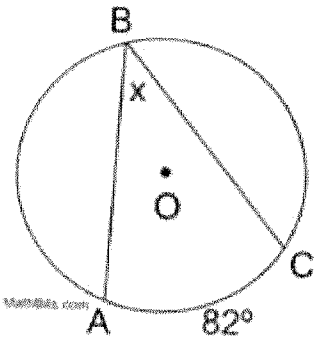
$$m\angle AOB = m\overset{\frown}{AB}$$

Example:

In the diagram on the right, ∠AOB is a central angle with an intercepted minor arc from A to B. If the arc AB is 82°, then m∠AOB is also 82°.

2) Inscribed Angle

An inscribed angle is an angle with its vertex "on' the circle, formed by two intersecting chords.



Inscribed Angle = $\frac{1}{2}$ Intercepted Arc

$$m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$$

Example:

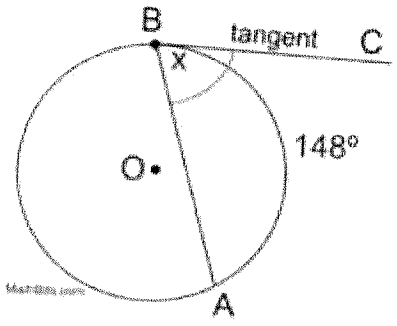
In the diagram at the right, ∠ABC is an inscribed angle with an intercepted minor arc from A to C. If the arc AC is 82°, then m∠AOB is 41°.

3) Tangent Chord Angle

A tangent chord angle is an angle formed by an intersecting tangent and chord that has its vertex on the circle.

Tangent Chord Angle = $\frac{1}{2}$ Intercepted Arc

$$m\angle ABC = \frac{1}{2}(m\overset{\frown}{AB})$$



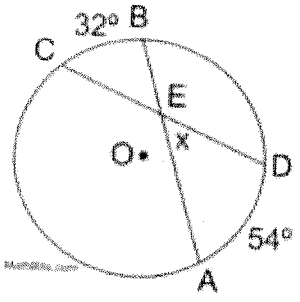
Example:
In the diagram on the right, ∠ABC is an angle formed by a tangent and chord with an intercepted minor arc from A to B. If the measure of the intercepted arc AB is 148°, then m∠ABC is 74°.

4) Angle Formed by Two Intersecting Chords

When two chords intersect inside a circle, four angles are formed. At the point of intersection, two sets of congruent vertical angles are formed in the corners of the X that appears.

Angle Formed by Two Chords
= $\frac{1}{2}$ (**SUM** of Intercepted Arcs)

$$m\angle AED = \frac{1}{2}(m\overset{\frown}{AD} + m\overset{\frown}{CB})$$



**Once you have found one of these angles, you automatically know the sizes of the other three by using vertical angles (which are congruent) and adjacent angles (which are supplementary).

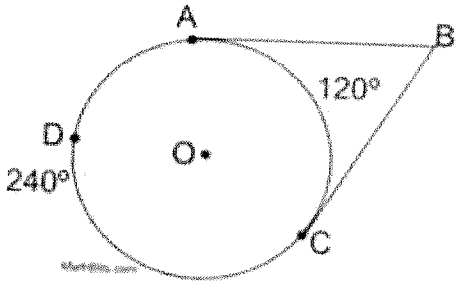
Example:
In the diagram at the right, ∠AED is an angle formed by two intersecting chords in the circle. Notice that the intercepted arcs belong to the set of vertical angles. Since we know that the arcs AD and CB are 54° and 32° respectively, using the given formula, we find that m∠AED is 43°

5) Angle Formed Outside of Circle by Intersection

Angles formed outside of a circle can be formed by the intersection of two tangent, two secants, or a tangent and a secant. The formula for each of these types of intersections is the same.

> Angle Formed by Two Tangents
> $= \frac{1}{2}$ (**DIFFERENCE** of Intercepted Arcs)
>
> $$m\angle ABC = \frac{1}{2}(m\overset{\text{major}}{\overset{\frown}{ADC}} - m\overset{\text{minor}}{\overset{\frown}{AC}})$$
>
> (When subtracting, start with the larger arc.)

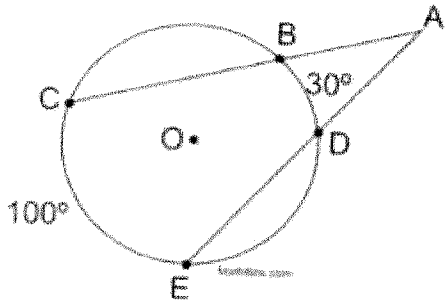
Example 1: Two tangents



$$m\angle ABC = \frac{1}{2}(240° - 120°)$$

$$= \frac{1}{2}(120°) = 60°$$

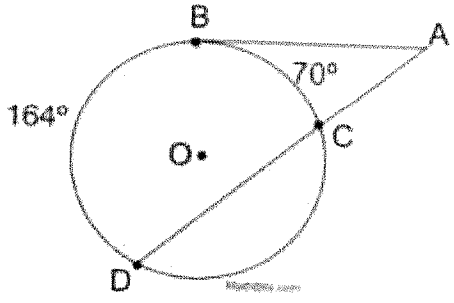
Example 2: Two secants



$$m\angle CAE = \frac{1}{2}(100° - 30°)$$

$$= \frac{1}{2}(70°) = 35°$$

Example 3: A tangent and a secant



$$m\angle BAD = \frac{1}{2}(164° - 70°)$$

$$= \frac{1}{2}(94°) = 47°$$