9. **a.** Horizontal change:

$$\frac{1}{4}|0-4| = \frac{1}{4}(-4)$$
$$= -1$$

Vertical change:

$$\frac{1}{4}|-7-(-5)| = \frac{1}{4}(-2)$$
$$= -\frac{1}{2}$$

Point

$$\left(4+(-1), -5+\left(-\frac{1}{2}\right)\right) = \left(3, \frac{-11}{2}\right)$$

b. For points (*a*, *b*) and (*c*, *d*), $K\left(a+\frac{c-a}{n}, b+\frac{d-b}{n}\right)$

10. Answers may vary. Sample: The student did not add the *x*- and *y*-coordinates when calculating the midpoint. The correct answer is $M\left(\frac{-4+(-1)}{2}, \frac{5+(-4)}{2}\right)$ or $M\left(\frac{-5}{2}, \frac{1}{2}\right)$.

11. Use the midpoint formula to set up simultaneous equations in two variables and solve for *a*.

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Let $(x_1, y_1) = (2a, 3b+3)$ and $(x_2, y_2) = (b+1, a+2)$.

Write an equation using the x -coordinate.

$$\frac{2a+(b+1)}{2} = 3$$
$$2a+b+1 = 6$$
$$2a+b = 5$$

Write an equation using the y -coordinate.

$$\frac{(3b+3)+(a+2)}{2} = 5$$
$$3b+3+a+2 = 10$$
$$a+3b = 5$$

Solve the simultaneous equations.

$$2a+b = 5$$
$$a+3b = 5$$

Multiply the first equation by 3 and subtract.

$$6a+3b = 15$$
$$a+3b = 5$$
$$5a = 10 \rightarrow a = 2$$

$a = 2$. Sample: Set up simultaneous equations in two variables and solve for *a*.

$$\frac{2a+b+1}{2} = 3, \ \frac{a+3b+5}{2} = 5 \rightarrow a = 2$$

12. **a.** Substitute the given information into the midpoint formula, and solve for the unknown values.

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Let $(x_1, y_1) = P(0, 0)$ and $(x_2, y_2) = Q$. Substitute.

$$(2, 5) = \left(\frac{0+x_2}{2}, \frac{0+y_2}{2}\right)$$
$$(2, 5) = \left(\frac{x_2}{2}, \frac{y_2}{2}\right)$$

Set up and solve an equation for each coordinate.

x-coordinate	*y*-coordinate
$2 = \frac{x_2}{2}$	$5 = \frac{y_2}{2}$
$x_2 = 4$	$y_2 = 10$

So, $Q = (4, 10)$.

b. Answers may vary. Sample: Find the point 4 times the distance that the point (2, 5) is from the origin by multiplying the *x*-coordinate and *y*-coordinate by 4: $Q(8, 20)$.

13. You use the distance formula to find the length of a line segment. If you square both sides of the equation, you get a version of the Pythagorean Theorem.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

This means we can use the Pythagorean Triple 8, 15, 17 to find candidates for Q.

Now substitute $P(-4, 7) = (x_1, y_1)$ into the equation and consider the resulting cases.

$17^2 = (x_2 - (-4))^2 + (y_2 - 7)^2$

Case 1:

$(x_2 + 4)^2 = 8^2$ with $x_2 > -4$ and $(y_2 - 7)^2 = 15^2$ with $y_2 > 7$.

$$(x_2 + 4)^2 = 8^2$$
$$x_2 + 4 = \pm 8$$

$x_2 + 4 = 8$, $x_2 = 4$ or $x_2 + 4 = -8$, $x_2 = -12$

Since $x_2 > -4$, $x_2 = 4$

$$(y_2 - 7)^2 = 15^2$$
$$y_2 - 7 = \pm 15$$

$y_2 - 7 = 15$, $y_2 = 22$ or $y_2 - 7 = -15$, $y_2 = -8$

Since $y_2 > 7$, $y_2 = 22$.

Case 2:

$(x_2 + 4)^2 = 15^2$ with $x_2 > -4$ and $(y_2 - 7)^2 = 8^2$ with $y_2 > 7$.

$$(x_2 + 4)^2 = 15^2$$
$$x_2 + 4 = \pm 15$$

$x_2 + 4 = 15$, $x_2 = 11$ or $x_2 + 4 = -15$, $x_2 = -19$

Since $x_2 > -4$, $x_2 = 11$.

$$(y_2 - 7)^2 = 8^2$$
$$y_2 - 7 = \pm 8$$

$y_2 - 7 = 8$, $y_2 = 15$ or $y_2 - 7 = -8$, $y_2 = -1$

Since $y_2 > 7$, $y_2 = 15$.

So, the possible values of are $Q(4, 22)$ and $Q(11, 15)$.

$(4, 22)$ and $(11, 15)$; answers may vary. Sample: Using the Pythagorean Triple 8, 15, 17, where 17 is the straight line distance, 8 is the horizontal distance, and 15 is the vertical distance, add 8 to -4 and 15 to 7 to get $(4, 22)$. Switch 8 and 15 to get $(11, 15)$.

14. Use the distance formula to find PM.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Let $(x_1, y_1) = M(c, d)$ and $(x_2, y_2) = P(a, b)$. Substitute.

$$PM = \sqrt{(a - c)^2 + (b - d)^2}$$

By the definition of the midpoint, $PM = \frac{1}{2}PQ$. Solve the equation for PQ and then substitute.

$$\begin{aligned} \tfrac{1}{2}PQ &= PM \\ PQ &= 2PM \\ PQ &= 2\sqrt{(a - c)^2 + (b - d)^2} \end{aligned}$$

$$PM = \sqrt{(a - c)^2 + (b - d)^2}$$

$$PQ = 2\sqrt{(a - c)^2 + (b - d)^2}$$

15. To find the point $\frac{3}{10}$ of the way from A to B, first find the horizontal and vertical changes from point A:

Horizontal change:

$$\begin{aligned} &\tfrac{3}{10}(12 - (-4)) = \tfrac{3}{10}(16) \\ &= 4.8 \end{aligned}$$

Vertical change:

$$\begin{aligned} &\tfrac{3}{10}(5 - (-7)) = \tfrac{3}{10}(12) \\ &= 3.6 \end{aligned}$$

Add the horizontal and vertical changes to the coordinates of point A:

$$(-4 + 4.8, -7 + 3.6) = (0.8, -3.4)$$

16. To find the point $\frac{1}{4}$ of the way from B to A, first find the horizontal and vertical changes from point B:

Horizontal change:

$$\begin{aligned} &\tfrac{1}{4}(-4 - 12) = \tfrac{1}{4}(-16) \\ &= -4 \end{aligned}$$

Vertical Distance:

$$\begin{aligned} &\tfrac{1}{4}(-7 - 5) = \tfrac{1}{4}(-12) \\ &= -3 \end{aligned}$$

Add the horizontal and vertical changes to the coordinates of point B:

$$(12 + (-4), 5 + (-3)) = (8, 2)$$

17. Use the midpoint formula $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ to find the midpoint of $\overline{PQ}$.

$$M = \left(\frac{-2+3}{2}, \frac{13+5}{2}\right)$$
$$= \left(\frac{1}{2}, 9\right)$$

18. Use the midpoint formula $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ to find the midpoint of $\overline{PQ}$.

$$M = \left(\frac{-2+1.4}{2}, \frac{2.5+4}{2}\right)$$
$$= (-0.3, 3.25)$$

19. Use the midpoint formula $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ to find the midpoint of $\overline{PQ}$.

$$M = \left(\frac{4\frac{1}{3}+\left(-2\frac{1}{5}\right)}{2}, \frac{3\frac{1}{6}+3\frac{2}{6}}{2}\right)$$
$$= \left(\frac{(60+5)-(30+3)}{30}, \frac{(18+1)+(18+2)}{12}\right)$$
$$= \left(\frac{32}{30}, \frac{39}{12}\right)$$
$$= \left(1\frac{1}{15}, 3\frac{5}{12}\right)$$

20. In order to find how far apart Arthur and Jaime are, we must first look at the grid and find their locations. Arthur is at located at $(20, 35)$, and Jaime is located at $(45, 20)$. We then apply the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, to find how far apart they are.

$$= \sqrt{(45-20)^2 + (20-35)^2}$$
$$= \sqrt{25^2 + (-15)^2}$$
$$= \sqrt{625 + 225}$$
$$= \sqrt{850}$$
$$\approx 29.2$$

Arthur and Jamie are about 29.2 m apart.

21. In order to find out who is closer to Cameron, we must first look at the grid and find their locations. Arthur is at located at $(20, 35)$, Jaime is located at $(45, 20)$, and Cameron is located at $(65, 40)$. We then apply the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, to find how far Arthur is from Cameron, and Jaime from Cameron, and then compare distances to see who is closer.

$AC = \sqrt{(65-20)^2 + (40-35)^2}$	$JC = \sqrt{(65-45)^2 + (40-20)^2}$
$= \sqrt{45^2 + 5^2}$	$= \sqrt{20^2 + 20^2}$
$= \sqrt{2025 + 25}$	$= \sqrt{400 + 800}$
$= \sqrt{2050}$	$= \sqrt{800}$
≈ 45.3	≈ 28.3

$45.3 \geq 28.3$

Jamie is closer.

22. In order to find out who is closest to the ball, we must first look at the grid and find their locations. Arthur is at located at $(20, 35)$, Jaime is located at $(45, 20)$, and Cameron is located at $(65, 40)$. The soccer ball is located at $(35, 60)$. We then apply the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, to find how the three people are from the ball and determine who is closest.

$$\begin{aligned} AB &= \sqrt{(35-20)^2 + (60-35)^2} \\ &= \sqrt{15^2 + 25^2} \\ &= \sqrt{225 + 625} \\ &= \sqrt{850} \\ &\approx 29.2 \end{aligned}$$

$$\begin{aligned} JB &= \sqrt{(35-45)^2 + (60-20)^2} \\ &= \sqrt{(-10)^2 + 40^2} \\ &= \sqrt{100 + 1600} \\ &= \sqrt{1700} \\ &\approx 41.2 \end{aligned}$$

$$\begin{aligned} CB &= \sqrt{(35-65)^2 + (60-40)^2} \\ &= \sqrt{(-30)^2 + 20^2} \\ &= \sqrt{900 + 400} \\ &= \sqrt{1300} \\ &\approx 36.1 \end{aligned}$$

Arthur is closest at about 29.2 m, Cameron is in the middle at about 36.1 m, and Jamie is farthest at about 41.2 m.

23. $\left(\frac{19}{3}, 5\right)$; Given that the new student center is $\frac{2}{3}$ the distance from the arts center to the residential complex. In order to find the coordinates of the new center, we must first look at the grid and find the locations of the other buildings. The arts center is located at $(1, 9)$ and the residential housing is located at $(9, 3)$. To find the point $\frac{2}{3}$ of the way from the arts center to the residential complex, first find the horizontal and vertical changes.

Horizontal change:

$$\frac{2}{3}(9-1) = \frac{2}{3}(8) = \frac{16}{3}$$

Vertical change:

$$\frac{2}{3}(3-9) = \frac{2}{3}(-6) = -4$$

Add the horizontal and vertical changes to the coordinates of the Arts Center:

$$\left(1 + \frac{16}{3}, 9 + (-4)\right) = \left(\frac{19}{3}, 5\right).$$

24. To find the area that the light covers, we must find the distance between the lighthouse and the pier. Applying the distance formula, we have:

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ d &= \sqrt{(9-2)^2 + (7-4)^2} \\ &= \sqrt{7^2 + 3^2} \\ &= \sqrt{49 + 9} \\ &= \sqrt{58} \\ &\approx 7.6158 \end{aligned}$$

$7.6158 = (7.6158)(.4) = 3.04632$ miles.

The distance of the beam of light is the value of the radius of light around the lighthouse. Use the formula for the area of a circle to solve for the area that the light covers.

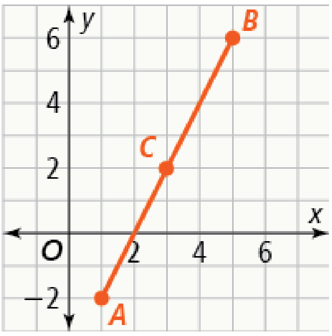
$$A = \pi r^2 = \pi(3.04632^2) \approx 29.15 \text{ mi}^2$$

25. In order for the ship captain to communicate with the deep-sea diver, the distance between the two must be less than 60 meters. Find the distance between the two using the distance formula and compare.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(48 - 8)^2 + (-36 - 8)^2}$$
$$= \sqrt{40^2 + 44^2}$$
$$= \sqrt{1600 + 1936}$$
$$= \sqrt{3536}$$
$$\approx 59.5$$

Yes; since $59.5 \leq 60$, the ship captain can communicate with the deep-sea diver.

26.



Graph the endpoint (A) and then the midpoint (C). Use the same rise and run from the endpoint to the midpoint but start at the midpoint to find the other endpoint (B).

27. (D): $17 \neq 24$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(23 - 6)^2 + (13 - (-4))^2}$$
$$= \sqrt{17^2 + 17^2}$$
$$= \sqrt{289 + 289}$$
$$= \sqrt{578}$$
$$\approx 24.0$$

28. **Part A** On Broadway, the parade travels about 6.71 units. On Central Avenue, the parade travels about 5.66 units. The total distance is 12.37 units, and according to the grid,

$12.37 \times 0.25 = 3.10$ miles. Therefore, the route exceeds 3 miles.

Part B Broadway and Cedar St.; from Broadway and Cedar St., the distance to the finish can be found by applying the distance formula to calculate the distance traveled along Broadway and Central Ave.

$$\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
d &= \sqrt{(6 - 10)^2 + (0 - 2)^2} \\
&= \sqrt{(-4)^2 + (-2)^2} \\
&= \sqrt{16 + 4} \\
&= \sqrt{20} \\
&\approx 4.47
\end{aligned}$$

$$\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
d &= \sqrt{(7 - 6)^2 + (4 - 0)^2} \\
&= \sqrt{1^2 + 4^2} \\
&= \sqrt{1 + 16} \\
&= \sqrt{17} \\
&\approx 4.12
\end{aligned}$$

The total distance is about $4.47 + 4.12 = 8.59$ units, which is about $8.59 \times 0.25 = 2.14$ miles. Therefore, the route will not exceed 3 miles.

Part C Applying the Midpoint Formula, we find the halfway point down each road of the parade.

$$\begin{aligned}
M &= \left(\tfrac{2+6}{2}, \tfrac{4+0}{2}\right) \\
&= (4, 2)
\end{aligned}
\qquad
\begin{aligned}
M &= \left(\tfrac{6+10}{2}, \tfrac{0+2}{2}\right) \\
&= (8, 1)
\end{aligned}$$