

9. By definition, the length of a segment is the absolute value of the difference of coordinates of the endpoints.

$$MN = |M - N|$$

$$MN = M - N \quad -MN = M - N$$

$$12 = 11 - N \quad \text{or} \quad -12 = 11 - N$$

$$N = -1 \quad N = 23$$

So, $N = -1$ or $N = 23$.

10. By repeatedly applying the segment addition property to the segments on the line, we find the following equations.

$$AC = AB + BC$$

$$AD = AC + CD$$

$$AE = AD + DE$$

Now substitute as needed.

$$AE = AD + DE$$

$$AE = AC + CD + DE$$

$$AE = AB + BC + CD + DE$$

11. By the Segment Addition Postulate, we know that $AB + BC = CD$ if A , B , and C are points on a line with B between A and C . We don't know if C is between D and E , so there are 2 cases that need to be considered.

If C is between D and E , we have $CD + CE = DE$. Substitute the known values and solve.

$$CD + CE = DE$$

$$20 + 32 = DE$$

$$52 = DE$$

If C is not between D and E , we have $CE + ED = CD$ or $CE + DE = CD$. Substitute the known values and solve.

$$CD + DE = CE$$

$$20 + DE = 32$$

$$DE = 12$$

So, $DE = 12$ or $DE = 52$.

12. Answers may vary. Sample: Benito used equal signs where he should have used congruency symbols. The segments and angles are congruent, but their lengths and measures are equal.
13. Opposite rays are rays with the same endpoint that lie on the same line, so $m\angle XYZ = 180^\circ$.

By the Angle Addition Postulate, $m\angle XWY + m\angle YWZ = m\angle XWZ$.

It's given that $m\angle XWY = 4(m\angle YWZ)$.

Substitute the given information and solve.

$$m\angle XWY + m\angle YWZ = m\angle XWZ$$

$$4(m\angle YWZ) + m\angle YWZ = 180^\circ$$

$$5(m\angle YWZ) = 180^\circ$$

$$m\angle YWZ = 36^\circ$$

So, $m\angle YWZ = 36^\circ$.

14. The area of the shaded region is the area of $BCFE$ plus the area of $DEHG$. To find the areas of these rectangles, find BC , BE , DG , and DE . By inspection, $AD \cong BE$ and $AB \cong DE$, so start by finding AB and AD .

In the diagram, $AB \cong AD$ which means that $ABED$ is a square.

$$AB^2 = 49$$

$$AB = 7$$

Use $AB = AD = 7$, the given information, and the Segment Addition Postulate to find DG and BC .

$$AD + DG = AG \quad AB + BC = AC$$

$$7 + DG = 9 \quad 7 + BC = 10$$

$$DG = 2 \quad BC = 3$$

Now find the areas.

Area $BCFE$:

$$A = BC \times BE$$

$$= 3 \times 7$$

$$= 21$$

Area $DEHG$:

$$A = DG \times DE$$

$$= 2 \times 7$$

$$= 14$$

So, the area of the shaded region is the area $BCFE$ plus the area of $DEHG$.

$$A = 21 + 14$$

$$= 35$$

Now, find the area of $ACIG$.

$$A = AC \times AG$$

$$= 10 \times 9$$

$$= 90$$

Now find the fraction.

$$\frac{\text{Area of Shaded Region}}{\text{Area of } ACIG} = \frac{35}{90} = \frac{7}{18}$$

So, the shaded region is $\frac{7}{18}$ of the area of $ACIG$.

15. Looking at the diagram, we can see the following congruencies.

$$\angle KNM \cong \angle JKN$$

$$\angle JNK \cong \angle NMK \cong \angle KML$$

Using the Angle Addition Postulate, we can relate the information given in the problem to these congruencies to set up and solve equations to find $m\angle NKM$.

$$m\angle LMK + m\angle KMN = m\angle LMN$$

$$m\angle KMN + m\angle KMN = 116^\circ$$

$$m\angle KMN + m\angle KMN = 116^\circ$$

$$2(m\angle KMN) = 116^\circ$$

$$m\angle KMN = 58^\circ$$

$$m\angle JNK + m\angle KNM = m\angle JNM$$

$$58^\circ + m\angle KNM = 103^\circ$$

$$m\angle KNM = 45^\circ$$

Therefore, $m\angle NKM = 77^\circ$.

16. $DF = |-4 - (-1)|$ $DF = |-1 - (-4)|$
 $= |-3|$ or $= |3|$
 $= 3$ $= 3$

So, $DF = 3$.

17. $DE = \left| -4 - \left(-1\frac{1}{3}\right) \right|$ $DE = \left| -1\frac{1}{3} - (-4) \right|$
 $= \left| -2\frac{2}{3} \right|$ or $= \left| 2\frac{2}{3} \right|$
 $= 2\frac{2}{3}$ $= 2\frac{2}{3}$

So, $DE = 2\frac{2}{3}$.

18. $FG = |-1 - 2|$ $FG = |2 - (-1)|$
 $= |-3|$ or $= |3|$
 $= 3$ $= 3$

So, $FG = 3$.

$$\begin{aligned}
 19. \quad FH &= \left| -1 - 3\frac{1}{2} \right| & FH &= \left| 3\frac{1}{2} - (-1) \right| \\
 &= \left| -4\frac{1}{2} \right| & \text{or} &= \left| 4\frac{1}{2} \right| \\
 &= 4\frac{1}{2} & &= 4\frac{1}{2}
 \end{aligned}$$

$$\text{So, } FH = 4\frac{1}{2}.$$

$$\begin{aligned}
 20. \quad GH &= \left| 2 - 3\frac{1}{2} \right| & GH &= \left| 3\frac{1}{2} - 2 \right| \\
 &= \left| -1\frac{1}{2} \right| & \text{or} &= \left| 1\frac{1}{2} \right| \\
 &= 1\frac{1}{2} & &= 1\frac{1}{2}
 \end{aligned}$$

$$\text{So, } GH = 1\frac{1}{2}.$$

$$\begin{aligned}
 21. \quad EH &= \left| -1\frac{1}{3} - 3\frac{1}{2} \right| & EH &= \left| 3\frac{1}{2} - \left(-1\frac{1}{3}\right) \right| \\
 &= \left| 4\frac{5}{6} \right| & \text{or} &= \left| -4\frac{5}{6} \right| \\
 &= 4\frac{5}{6} & &= 4\frac{5}{6}
 \end{aligned}$$

$$\text{So, } EH = 4\frac{5}{6}.$$

22. Use the Segment Addition Postulate and the segment lengths from the diagram to find x .

$$\begin{aligned}
 AD + BC &= AC \\
 x + 7 + 2x &= 16 \\
 3x &= 9 \\
 x &= 3
 \end{aligned}$$

23. Recall that in exercise 22, we found that $x = 3$. Substitute for x in the expression for AB .

$$\begin{aligned}
 AB &= x + 7 \\
 AB &= 10
 \end{aligned}$$

24. Use the Segment Addition Postulate and the segment lengths from the diagram to find BD . Recall that in exercise 22, we found that $x = 3$.

$$\begin{aligned}
 BD &= BC + CD \\
 &= 2x + 3x - 1 \\
 &= 6 + 9 - 1 \\
 &= 14
 \end{aligned}$$

$$\text{So, } BD = 14.$$

25. Use the Segment Addition Postulate and the segment lengths from the diagram to find CE . Recall that in exercise 22, we found that $x = 3$.

$$\begin{aligned}CE &= CD + DE = CE \\&= 3x - 1 + 2x + 3 \\&= 9 - 1 + 6 + 3 \\&= 17\end{aligned}$$

So, $CE = 17$.

26. By the Angle Addition Postulate, $m\angle POQ + m\angle QOR = m\angle POR$. Substitute the given angle measures into the equation and solve for $m\angle QOR$.

$$\begin{aligned}m\angle POQ + m\angle QOR &= m\angle POR \\24^\circ + m\angle QOR &= 59^\circ \\m\angle QOR &= 35^\circ\end{aligned}$$

So, $m\angle QOR = 35^\circ$.

27. Use the Angle Addition Postulate and substitution to write $m\angle POS$ as the sum of the angles with known measurements. Then substitute the angle measurements into the resulting equation to find $m\angle POS$.

$$\begin{aligned}m\angle POQ + m\angle QOR &= m\angle POR \\m\angle POR + m\angle ROS &= m\angle POS \\m\angle POS &= m\angle POQ + m\angle QOR + m\angle ROS \\&= 19^\circ + 31^\circ + 15^\circ \\&= 65^\circ\end{aligned}$$

So, $m\angle POS = 65^\circ$.

28. Use the Angle Addition Postulate to write equations relating the given and desired angle measurements.

$$m\angle POQ + m\angle QOR = m\angle POR$$

$$m\angle QOR + m\angle ROS = m\angle QOS$$

Substitute the known values into the first equation and find $m\angle QOR$.

$$m\angle POQ + m\angle QOR = m\angle POR$$

$$28^\circ + m\angle QOR = 61^\circ$$

$$m\angle QOR = 33^\circ$$

Now, substitute for the known values in the second equation to find $m\angle ROS$.

$$m\angle QOR + m\angle ROS = m\angle QOS$$

$$33^\circ + m\angle ROS = 46^\circ$$

$$m\angle ROS = 13^\circ$$

So, $m\angle ROS = 13^\circ$

29. Looking at the diagram, we see that $\overline{EF} \cong \overline{EG}$. Therefore, $EF = EG = 3$.
30. By the Segment Addition Postulate, $AG = AE + EG$. Looking at the diagram, we see that $\overline{AE} \cong \overline{EB}$, so $AE = 8$. Substitute the known lengths into the equation to find AG .

$$AG = AE + EG$$

$$= 8 + 3$$

$$= 11$$

Therefore, $AG = 11$.

31. By the Segment Addition Postulate, $AD + DF = AF$. Looking at the diagram, we see that $\overline{DF} \cong \overline{EG}$, so $DF = 3$. Substitute the known lengths into the equation to find AD .

$$AD + DF = AF$$

$$AD + 3 = 7$$

$$AD = 4$$

Therefore, $AD = 4$.

32. Looking at the diagram, we see that $\angle EFG \cong \angle EGF$. Therefore, $m\angle EFG = m\angle EGF = 28^\circ$.

33. By the Angle Addition Postulate, $m\angle CAF + m\angle FAE = m\angle CAE$. Looking at the diagram, we see that $\angle FAE \cong \angle EBG$, so $m\angle FAE = 19^\circ$. Substitute the known angle measures into the equation to find $m\angle CAF$.

$$m\angle CAF + m\angle FAE = m\angle CAE$$

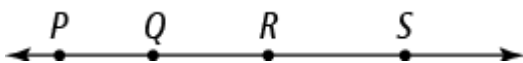
$$m\angle CAF + 19^\circ = 51^\circ$$

$$m\angle CAF = 32^\circ$$

Therefore, $m\angle CAF = 32^\circ$.

34. Looking at the diagram, we see that $\overline{DF} \cong \overline{EG}$. Therefore, $DF = EG = 3$.

35. Start by sketching a line and locating the points.



Next, use the Segment Addition Postulate and substitution to write and solve equations involving the known and desired segments.

$$PR + RS = PS$$

$$15 + RS = 18$$

$$RS = 3$$

$$PQ + QR = 15$$

$$RS + QR = 15$$

$$QR = 12$$

Therefore, $QR = 12$.

36. Springfield; Sample answer: Since the distance to Gilmore is 26 mi, halfway is 13 mi. Springfield is 15 mi away, so it is 2 mi away from being halfway, $|15 - 13| = 2$. Green Lake is 9 mi away, so it is 4 mi away from being halfway, $|9 - 13| = 4$. Since $2 < 4$, Springfield is closest to the halfway point.

37. To decide if the building should be approved, find the height of the building. The height will be the sum of the heights of the floors.

$$h = 20 + 15 \times 11$$

$$= 185$$

Since $185 < 310$, the height of the building is below the height limit and the city planning commission should approve the building.

Yes; Sample answer: According to the plan, the building will have a height of 185 ft, which is below the height limitation for the area.

- 38.** To find the number of trees that will be planted, find the length of Dayton Avenue in front of the plot of land. The perimeter of the figure is the sum of the side lengths of the figure. Notice how the length of Dayton Avenue is split into two segments where each segment is congruent to a side of the figure. This means that the sum of the lengths of the congruent segments is twice the length of Dayton Avenue. Let D be the length of Dayton Avenue, and write an equation for the perimeter of the plot of land. Then substitute the known values and solve for D .

$$52 + 68 + 2D = P$$

$$120 + 2D = 234$$

$$2D = 114$$

$$D = 57$$

To find the number of trees, divide 57 by 20.

$$57 \div 20 = 2.85$$

2.85 is close to 3, so the city will plant about 3 trees.

3; Sample: Since $234 - 52 - 68 = 114$, the total length of the four segments with tick marks is 114 feet. There are two segments with one tick mark and two sides with two tick marks, and the length of Dayton Avenue is the sum of the length of a segment with one tick mark and the length of a segment with two tick marks. The total length along Dayton Avenue is half of 114 feet, or 57 feet. $57 \div 20 \approx 60 \div 20 = 3$.

39. Use the Segment Addition Postulate and the given information to determine if each equation is true.

A. True;

$$FH = 2FG \quad \text{Given}$$

$$FH = FG + GH \quad \text{Segment Addition Postulate}$$

$$2FG = FG + GH \quad \text{Substitution}$$

$$FG = GH \quad \text{Subtraction Property of Equality}$$

$$GH = HI \quad \text{Given}$$

$$FG = HI \quad \text{Substitution}$$

B. There is no information given about segments containing J , so nothing can be proven or disproven about statements involving J .

C. True;

$$FH = 2FG, GH = HI, FI = IK \quad \text{Given}$$

$$FI = FH + HI \quad \text{Segment Addition Postulate}$$

$$FI = 2FG + GH \quad \text{Substitution}$$

$$IK = 2FG + GH \quad \text{Subtraction Property of Equality}$$

$$FH = FG + GH \quad \text{Segment Addition Postulate}$$

$$2FG = FG + GH \quad \text{Substitution}$$

$$FG = GH \quad \text{Subtraction Property of Equality}$$

$$IK = 2FG + FG \quad \text{Substitution}$$

$$IK = 3FG$$

D. True;

$$GI = GH + HI \quad \text{Segment Addition Postulate}$$

$$FG = HI \quad \text{Previously proved in Part A}$$

$$GI = GH + FG \quad \text{Substitution}$$

$$FH = FG + GH \quad \text{Segment Addition Postulate}$$

$$FH = GI \quad \text{Substitution}$$

E. There is no information given about segments containing J , so nothing can be proven or disproven about statements involving J .

F. True;

$$GI = GH + HI \quad \text{Segment Addition Postulate}$$

$$GH = HI \quad \text{Given}$$

$$GI = HI + HI \quad \text{Substitution}$$

$$GI = 2HI \quad \text{Addition Property of Equality}$$

$$2GI = 4HI \quad \text{Multiplication Property of Equality}$$

$$HK = HI + IK \quad \text{Segment Addition Postulate}$$

$IK = 3FG$ Previously Proved in Part C

$FG = HI$ Previously proved in Part A

$HK = FG + 3FG$ Substitution

$HK = 4FG$ Distributive Property

$HK = 4HI$ Substitution

$HK = 2GI$ Substitution

$2GI = HK$

$2GI = HI + HK$ Segment Addition Postulate

$2GI = HI + 3FG$ Previously proved in Part C

$2GI = HI + 3HI$ Previously proved in Part A

$2GI = 2(HI + HI)$ Distributive Property

$2GI = 2(GH + HI)$ Given $GH = HI$

$2GI = 2(GI)$ Segment Addition Postulate

40. Given $\angle ABC \cong \angle CBD$, then $m\angle ABC = m\angle CBD$. Substitute the measures of the angles and solve for x .

$$4x = \frac{5}{2}x + 18$$

$$\frac{3}{2}x = 18$$

$$x = 12$$

By the Angle Addition Postulate, $m\angle ABD = m\angle ABC + m\angle CBD$. Using the given information, we get $m\angle ABD = 2m\angle CBD$. Substitute for $m\angle CBD$ and x to find $m\angle ABD$.

$$m\angle ABD = 2m\angle CBD$$

$$= 2(4x)$$

$$= 8x$$

$$= 8(12)$$

$$= 96$$

So $m\angle ABD = 96^\circ$.

The correct answer is choice E.

41. Answers will vary. Students' floor plan should include four rooms, two walls of equal length, and two angles of equal measure, and their equations should show congruent angles and segments.