**10.** Prism $P$; To find the length of the diagonal, note that it is the hypotenuse of a right triangle with the height of the prism as one leg and the diagonal of the base as the other. Find the diagonal in each prism by applying the Pythagorean Theorem.

Prism $P$: Find $c$, the diagonal of the base. Let $l = 4$ and $w = 3$.

$$c^2 = l^2 + w^2$$
$$c^2 = 4^2 + 3^2$$
$$c^2 = 16 + 9$$
$$c = \sqrt{25}$$
$$c = 5$$

Now let $h = 15$ and find $d$, the diagonal of the prism.

$$d^2 = c^2 + h^2$$
$$d^2 = 5^2 + 15^2$$
$$d^2 = 250$$
$$d = 5\sqrt{10}$$

$$d = 5\sqrt{10} \approx 15.8$$

Prism $Q$: Let $l = 12$, $w = 4$, and $h = 9$. Find $c$ and $d$.

$$c^2 = l^2 + w^2$$
$$c^2 = 12^2 + 4^2$$
$$c^2 = 144 + 16$$
$$c = \sqrt{160}$$
$$c = 4\sqrt{10}$$

$$d^2 = c^2 + h^2$$
$$d^2 = \left(4\sqrt{10}\right)^2 + 9^2$$
$$d^2 = 160 + 81$$
$$d = \sqrt{241}$$

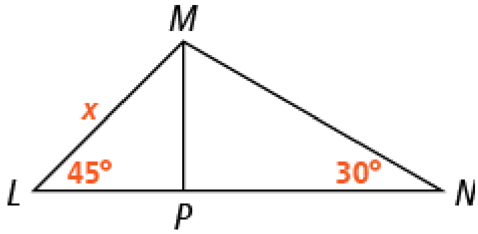
$$d = \sqrt{241} \approx 15.5$$

The diagonal of prism $P$ is longer than the diagonal of prism $Q$.

**11.** $\triangle DEF$ is an isosceles right triangle, so it is a $45^\circ$ - $45^\circ$ - $90^\circ$ triangle with hypotenuse of length 21. By Theorem 8-3, the length of the hypotenuse of a $45^\circ$ - $45^\circ$ - $90^\circ$ triangle is $\sqrt{2}$ times the length of the sides. Solve for $EF$.

$$\sqrt{2}EF = 14$$
$$EF = \frac{14}{\sqrt{2}}$$
$$EF = 7\sqrt{2}$$
$$EF \approx 9.9$$

**12.**




$MN = \sqrt{2}x$, $LN = \frac{\sqrt{2}}{2}x + \frac{\sqrt{6}}{2}x$; To find $MN$ and $LN$, note that $m\angle L = 45^\circ$ and $m\angle N = 30^\circ$. Start by drawing altitude $\overline{MP}$, where $P$ is on the line $\overline{LN}$. This divides $\triangle LMN$ into $\triangle LMP$ which is a $45^\circ$ - $45^\circ$ - $90^\circ$ triangle and $\triangle MNP$ which is a $30^\circ$ - $60^\circ$ - $90^\circ$ triangle.

$\overline{LM}$ is the hypotenuse of a $45^\circ$ - $45^\circ$ - $90^\circ$ triangle with legs $\overline{MP}$ and $\overline{LP}$. Find $MP$.

$$\sqrt{2}MP = LM$$
$$\sqrt{2}MP = x$$
$$MP = \frac{\sqrt{2}}{2}x$$

$\overline{MP}$ is the short leg of a $30^\circ$ - $60^\circ$ - $90^\circ$ triangle with long leg $\overline{NP}$ and hypotenuse $\overline{MN}$.

Find $MN$ and $PN$.

$$MN = 2MP \qquad PN = \sqrt{3}MP$$
$$MN = 2\left(\frac{\sqrt{2}}{2}x\right) \qquad PN = \sqrt{3}\left(\frac{\sqrt{2}}{2}x\right)$$
$$MN = \sqrt{2}x \qquad PN = \frac{\sqrt{6}}{2}x$$

To find $LN$, note that $LN = LP + PN$. Substitute. Remember, $LP = MP$.

$$LN = LP + PN$$
$$= \frac{\sqrt{2}}{2}x + \frac{\sqrt{6}}{2}x$$

So, $MN = \sqrt{2}x$ and $LN = \frac{\sqrt{2}}{2}x + \frac{\sqrt{6}}{2}x$.

**13.** If $\triangle XYZ$ is a right triangle, then $XY^2 + ZX^2 = YZ^2$ and $\sqrt{XY^2 + ZX^2} = YZ$.

If $XY^2 + ZX^2 > YZ^2$, then $\sqrt{XY^2 + ZX^2} > YZ$ and, by the Hinge Theorem, $m\angle X > 90^\circ$.

If $XY^2 + ZX^2 < YZ^2$, then $\sqrt{XY^2 + ZX^2} < YZ$ and, by the Hinge Theorem, $m\angle X < 90^\circ$.

**14.** By Theorem 8-4, a $30^\circ$ - $60^\circ$ - $90^\circ$ triangle has a hypotenuse that is twice the length of the short leg and a long leg that is $\sqrt{3}$ times the length of the short leg. Let the length of the hypotenuse be $h$. So the length of the short leg is $\frac{h}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}h$ and the length of the long leg is $\frac{2\sqrt{3}}{3}h$. This gives $JK = \frac{2\sqrt{3}}{3}KL$ or $KL = \frac{\sqrt{3}}{2}JK$.

**15.** $RS = 13$; Use the Pythagorean Theorem to find the length of the unknown side.

$$a^2 + b^2 = c^2$$

The missing side of the triangle is a leg. Let $a = 9$ and $c = 5\sqrt{10}$. Find $RS$.

$$c^2 = a^2 + b^2$$
$$\left(5\sqrt{10}\right)^2 = 9^2 + b^2$$
$$250 = 81 + b^2$$
$$b = \sqrt{169}$$
$$b = 13$$

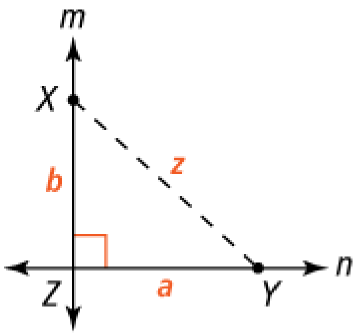
**16.** $XY = \sqrt{269}$; Use the Pythagorean Theorem to find the length of the unknown side.

$$a^2 + b^2 = c^2$$

The missing side of the triangle is the hypotenuse. Let $a = 10$ and $b = 13$. Find $XY$.

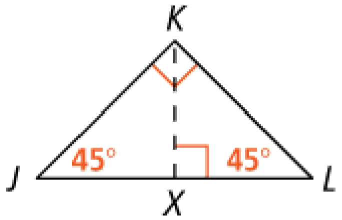
$$c^2 = a^2 + b^2$$
$$c^2 = 10^2 + 13^2$$
$$c^2 = 100 + 169^2$$
$$c = \sqrt{269}$$

**17.**




Draw perpendicular lines $m$ and $n$ intersecting at $Z$. On line $m$, plot $X$ so that $XZ = b$. On line $n$, plot $Y$ so that $YZ = a$. Draw $\overline{XY}$ with length $z$. By the Pythagorean Theorem, $a^2 + b^2 = c^2$. Given that $a^2 + b^2 = z^2$, by the Transitive Property of Equality, $c^2 = z^2$, so $c = z$. By SSS, $\triangle XYZ \cong \triangle ABC$, and $\angle X \cong \angle A$ by CPCTC. Therefore, $m\angle A = 90^\circ$ and $\triangle ABC$ is a right triangle.

**18.**



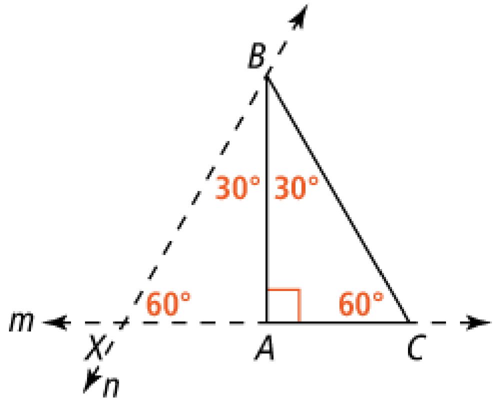

Given: $\triangle JKL$ with 45°,45°, and 90° angles

Draw altitude $\overline{KX}$.

Prove: $JL = \sqrt{2}\,JK$

| Statement | Reason |
|---|---|
| **1)** $\triangle JKL \sim \triangle JXK \sim \triangle LXK$ | **1)** Theorem 7-4 |
| **2)** $\frac{JL}{JK} = \frac{JK}{JX}$ | **2)** Corollary to Theorem 7-4 |
| **3)** $JX = \frac{1}{2}JL$ | **3)** The altitude from the vertex of an isosceles triangle bisects the noncongruent side. |
| **4)** $\frac{JL}{JK} = \frac{JK}{\frac{1}{2}JL}$ | **4)** Substitution |
| **5)** $(JL)^2 = 2(JK)^2$ | **5)** Multiplication Property of Equality |
| **6)** $JL = \sqrt{2}JK$ | **6)** Definition of square root |

**19.**




Prove:

1) $BC = 2AC$

2) $AB = AC\sqrt{3}$

Draw line $\boldsymbol{m}$ through $\overline{AC}$. Draw line $\boldsymbol{n}$ through $B$ and intersecting line $\boldsymbol{m}$ at a $60^\circ$ angle at point $X$ to form equilateral $\triangle XYZ$.

1) $\overline{AB}$ is an altitude of $\triangle XBC$ and bisects $\overline{XC}$. So, $XC = 2AC$. Because $\triangle XBC$ is equilateral, $XC = BC$ and $BC = 2AC$.

2) Use the Pythagorean Theorem.

$$(AC)^2 + (AB)^2 = (BC)^2$$
$$(AC)^2 + (AB)^2 = (2AC)^2$$
$$(AB)^2 = 3(AC)^2$$
$$AB = AC\sqrt{3}$$

**20.** $GJ = HJ = 6\sqrt{2}$; $\triangle GHJ$ is a $45^\circ$ - $45^\circ$ - $90^\circ$ triangle with hypotenuse of length 12 and legs with lengths $GJ$ and $HJ$. Apply Theorem 8-3 to find $GJ$.

$$GJ\sqrt{2} = 12$$
$$GJ = \frac{12}{\sqrt{2}}$$
$$GJ = \frac{12\sqrt{2}}{\sqrt{2}\sqrt{2}}$$
$$GJ = 6\sqrt{2}$$

Since $\triangle GHJ$ is an isosceles triangle, $GJ = HJ = 6\sqrt{2}$.

**21.** $XY = 9$, $YZ = 9\sqrt{3}$; $\triangle XYZ$ is a $30^\circ$ - $60^\circ$ - $90^\circ$ triangle with hypotenuse $\overline{ZX}$, long leg $\overline{YZ}$, and short leg $\overline{XY}$. Apply Theorem 8-4 to find $XY$ and $YZ$.

$$\begin{aligned} 2XY &= XZ \\ 2XY &= 18 \\ XY &= 9 \end{aligned} \quad \textbf{and} \quad \begin{aligned} YZ &= XY\sqrt{3} \\ YZ &= (9)\sqrt{3} \\ YZ &= 9\sqrt{3} \end{aligned}$$

So, $XY = 9$ and $YZ = 9\sqrt{3}$.

**22.** It is given that $\triangle QRT$ is a $30^\circ$ - $60^\circ$ - $90^\circ$ triangle and $\triangle RST$ is a $45^\circ$ - $45^\circ$ - $90^\circ$ triangle. Since $\overline{QS} = \overline{QT} + \overline{TS}$ and by Theorem 8-3 $TS = RT$, find $RT$ and then find $QS$.

$\overline{QT}$ is the long leg of a $30^\circ$ - $60^\circ$ - $90^\circ$ triangle with short leg $\overline{RT}$. Find $RT$.

$$\begin{aligned} \sqrt{3}RT &= QT \\ \sqrt{3}RT &= 10 \\ RT &= \frac{10}{\sqrt{3}} \\ RT &= \frac{10\sqrt{3}}{3} \end{aligned}$$

Find $QS$.

$$\begin{aligned} QS &= QT + TS \\ &= 10 + \frac{10\sqrt{3}}{3} \end{aligned}$$