12. By the Triangle Exterior Angle Theorem, the student should have set the exterior angle to equal the sum of the two remote interior angles, but the student did not use the correct angles. The student should have used $w^\circ = x^\circ + 31^\circ + 44^\circ$. Then $x^\circ = 49^\circ$.

14. First, solve for x°. Using the Alternate Interior Angle Theorem, we know that the angle directly to the left of x° has a value of 66°. Then use the Triangle Angle-Sum Theorem to set up an equation to solve for x°:

$$x + 66^\circ + 74^\circ = 180^\circ$$
$$x = 40^\circ$$

By the Vertical Angles Theorem, y° is congruent to the angle 66° and so $y = 66^\circ$.

By the Triangle Exterior Angle Theorem, z° is the exterior angle whose value is the sum of the two remote interior angles, x° and 66°. Set up the equation and solve for z°:

$$z^\circ = 66^\circ + x^\circ$$
$$z^\circ = 66^\circ + 40^\circ$$
$$z^\circ = 106^\circ$$

15. Use the Triangle Angle-Sum Theorem to write an equation using the sum of the values of the interior triangle angles and solve for x° Type equation here.:

$$x^\circ + (2x)^\circ + (3x)^\circ = 180^\circ$$
$$(6x)^\circ = 180^\circ$$
$$x^\circ = 30^\circ$$

Now substitute the value of x into the expressions for each labeled angle:

$$(5x)^\circ = (5 \cdot 30)^\circ = 150^\circ$$
$$(3x)^\circ = (3 \cdot 30)^\circ = 90^\circ$$
$$(2x)^\circ = (2 \cdot 30)^\circ = 60^\circ$$

16. Use the Triangle Angle-Sum Theorem to write and solve an equation for x°.

$$71^\circ + 46^\circ + x^\circ = 180^\circ$$
$$x^\circ = 63^\circ$$

17. Use the Triangle Angle-Sum Theorem to write and solve an equation for x°.

$$56^\circ + 76^\circ + x^\circ = 180^\circ$$
$$x^\circ = 48^\circ$$

18. Use the Triangle Angle-Sum Theorem to write and solve an equation for x°.

$46^\circ + 36^\circ + x^\circ = 180^\circ$
$x^\circ = 98^\circ$

By the Vertical Angles Theorem, x° is congruent to the missing angle value in the right side triangle. Use the Triangle Angle-Sum Theorem to write and solve an equation for y°.

$44^\circ + x^\circ + y^\circ = 180^\circ$
$44^\circ + 98^\circ + y^\circ = 180^\circ$
$y^\circ = 38^\circ$

19. Use the Triangle Exterior Angle Theorem to write and solve an equation for x°.

$x + 71^\circ = 91^\circ$
$x = 20^\circ$

Use the Triangle Angle-Sum Theorem to write and solve an equation for y°.

$39^\circ + 91^\circ + y^\circ = 180^\circ$
$y^\circ = 50^\circ$

20. Use the Triangle Exterior Angle Theorem to write and solve an equation for x°.

$x^\circ = 98^\circ + 41^\circ$
$x^\circ = 139^\circ$

21. Use the Triangle Exterior Angle Theorem to write and solve an equation for x°.

$x^\circ = 42^\circ + 79^\circ$
$x^\circ = 121^\circ$

22. Use the Triangle Exterior Angle Theorem to write and solve an equation for x°.

$x^\circ + 46^\circ = 119^\circ$
$x^\circ = 73^\circ$

23. Use the Triangle Exterior Angle Theorem to write and solve an equation for x°.

$x^\circ + 23^\circ = 74^\circ$
$x^\circ = 51^\circ$

24. Use the Triangle Angle-Sum Theorem to write and solve an equation for $m\angle 1$.

$m\angle 1 + 37^\circ + 116^\circ = 180^\circ$
$m\angle 1 = 27^\circ$

25. Notice that $\angle 2$ and the 74° angle form an exterior angle of the triangle that contains $\angle 1$. Use the Triangle Exterior Angle Theorem to find $m\angle 2$.

$74^\circ + m\angle 2 = 116^\circ + 37^\circ$
$m\angle 2 = 79^\circ$

26. Use the Triangle Angle-Sum Theorem to write and solve an equation for $m\angle 3$.

$48^\circ + 74^\circ + m\angle 3 = 180^\circ$
$m\angle 3 = 58^\circ$

27. Use the Triangle Exterior Angle Theorem to write and solve an equation for $m\angle 4$.

$m\angle 4 = 16^\circ + 122^\circ$
$m\angle = 138^\circ$

28. In an isosceles triangle, the angles opposite the triangle legs are congruent. Let $m\angle x$ be the measure of the angles opposite the legs. Let $\angle y$ be the third angle. Use the Triangle Angle-Sum Theorem to write an equation relating these three angles.

$$m\angle x + m\angle x + m\angle y = 180^\circ$$
$$2m\angle x + m\angle y = 180^\circ$$
$$m\angle y = 180^\circ - 2m\angle x$$

Since the angle is large, the stick must be fastened to a leg so that it extends from $\angle y$. This means that the remote interior angles are both equal to $m\angle x$. Use the Triangle Exterior Angle Theorem to write an equation to solve for $\angle x$.

$$m\angle x + m\angle x = 84^\circ$$
$$2m\angle x = 84^\circ$$
$$m\angle x = 42^\circ$$

Now substitute in the first equation and solve for $\angle y$.

$$m\angle y = 180^\circ - 2(42^\circ)$$
$$m\angle y = 96^\circ$$

So, the angles of the triangle measure to be 42°, 42° and 96°.

29. **a.** Notice that a right angle is exterior of the triangle that contains $\angle w$ and $\angle x$, and removed from the two angles. Use the Triangle Exterior Angle Theorem to relate $\angle w$ and $\angle x$ to the exterior angle.

$$w^\circ + x^\circ = 90^\circ$$

b. Notice that $\angle z$ is an exterior angle to the triangle that contains $\angle y$ and a right angle, and is removed from the two angles. Apply the Triangle Exterior Angle Theorem to find an equation that relates $\angle y$ and $\angle z$:

$$z^\circ = 90^\circ + y^\circ$$