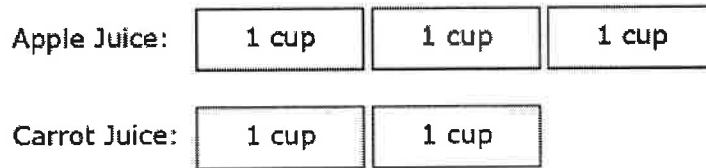


**Unit Rates (7.RP.1)**

1. This diagram shows how much apple juice is mixed with carrot juice for a recipe.



How many cups of apple juice are used for 1 cup of carrot juice? Show your work to find this unit rate and include units.

For a drink recipe, there is a ratio of 3 gallons mango juice to 12 gallons of peach juice.

How many gallons of mango juice are used for 1 gallons of peach juice? Show your work to find this unit rate and include units.

3. The train ride at the zoo covers a distance of  $3\frac{1}{4}$  miles in  $\frac{1}{3}$  of an hour.

How many miles per hour does the train go? Show your work to find this unit rate and include units.

## Unit Rates (7.RP.1)

Definition:

Unit Rate

A rate where the second number or denominator is 1.

Finding unit rates with Proportions (using scale factor):

- Set up a proportion where the first fraction is the original ratio of information, and the second fraction is  $x$  over 1—make sure the denominator is the unit being reduced to 1
- Find a scale factor from the denominator, and apply it to the numerator
- Include units with your unit rate, and make sure 1 is the second number

Example:

**A** FreshFoods has oranges on sale at 10 for \$2. For each part, find the unit rate. Be sure to label your answers with the proper units.

1. What is the cost per orange?

$$\frac{\$2}{10 \text{ oranges}} = \frac{x}{1 \text{ orange}} \quad x = 2 \cdot 0.1 = \$0.20 \text{ per 1 orange}$$

2. How many oranges can you buy for \$1?

$$\frac{10 \text{ oranges}}{\$2} = \frac{x}{\$1} \quad x = 10 \cdot 0.5 = 5 \text{ oranges per } \$1$$

Finding unit rates with Division:

- Set up a division problem where the second number is the unit being reduced to 1
- Include units with your unit rate, and make sure 1 is the second number

Example:

3. At FreshFoods, pasta is on sale at 6 boxes for \$5. Gus decides he needs to divide. What value does the quotient  $6 \div 5$  describe? What value does the quotient  $5 \div 6$  describe?

$$6 \text{ boxes} \div \$5 = 1.2 \text{ boxes per } \$1$$
$$\$5 \div 6 \text{ boxes} = \$0.83 \text{ per 1 box}$$

**Proportional Relationships (7.RP.2a)**

1. Circle ALL the tables below that represent a proportional relationship between  $x$  and  $y$ .

A.

$x$	$y$
1	4
2	7
3	10
4	13

B.

$x$	$y$
1	4
2	8
3	12
4	16

C.

$x$	$y$
1	6
2	7
3	8
4	9

D.

$x$	$y$
5	-2
7	0
9	2
11	4

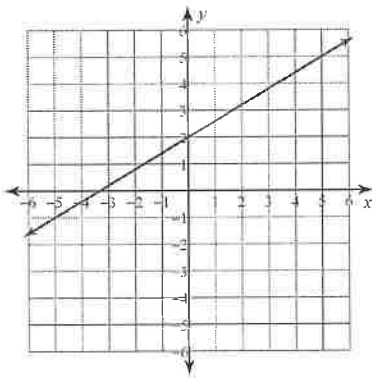
E.

$x$	$y$
3	12
5	20
2	8
8	32

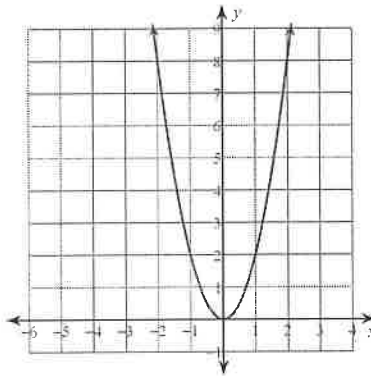
2. Explain how you know if a TABLE represents a proportional relationship.

3. Circle ALL the graphs below that show a proportional relationship between  $x$  and  $y$ .

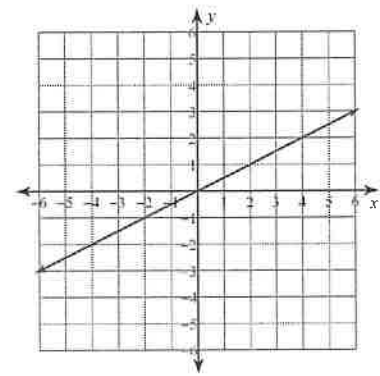
A.



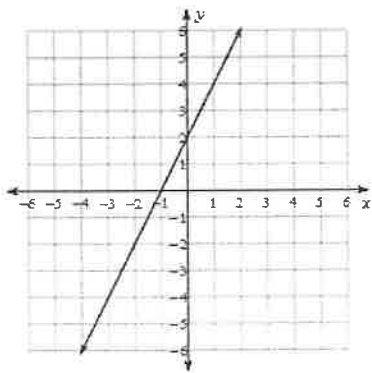
B.



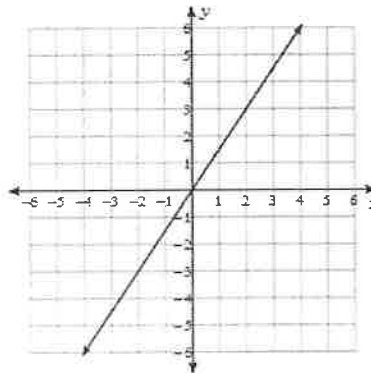
C.



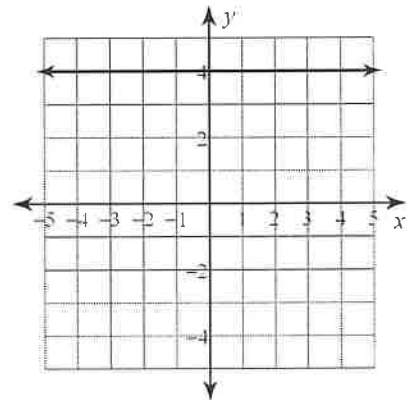
D.



E.



F.



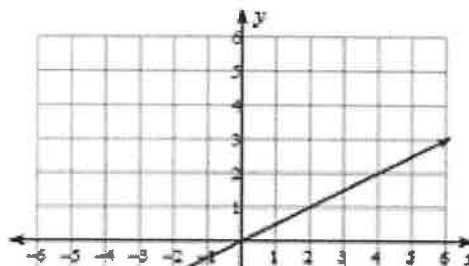
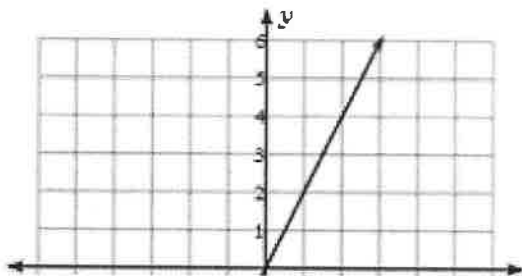
4. Explain how you know if a GRAPH represents a proportional relationship.

## Proportional Relationships (7.RP.2a)

### Proportional Relationships in a GRAPH:

- are a straight line
- go through (0, 0)

### Examples:



### Proportional Relationships in a TABLE:

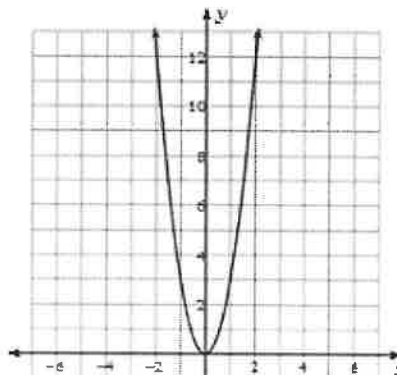
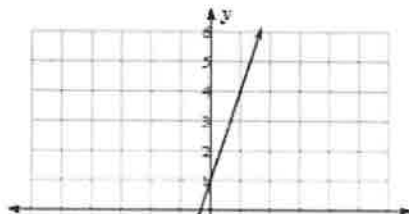
- have equivalent ratios between the values

### Example:

Number of Pizzas	1	2	3	4	5	10	15
Price for Pick Up	\$13	\$26	\$39	\$52	\$65	\$130	\$195

$$\frac{1}{13} = \frac{2}{26}$$

### NON-Examples: These are NOT proportional relationships:



Number of Pizzas	1	2	3	4	5	10	15
Price if Howdy's Delivers	\$18	\$31	\$44	\$57	\$70	\$135	\$200

$$\frac{1}{18} \neq \frac{2}{31}$$

**Constant of Proportionality and Proportional Equations (7.RP.2b, 7.RP.2c)**

1. The following tables show a proportional relationship between x and y.

A.

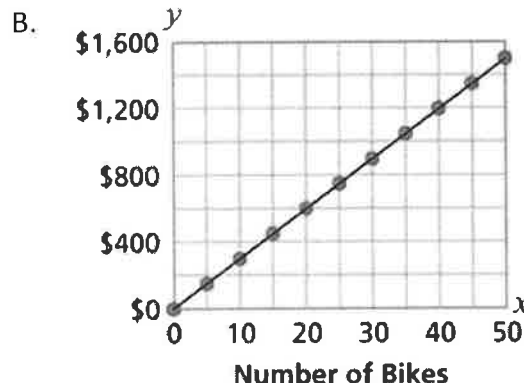
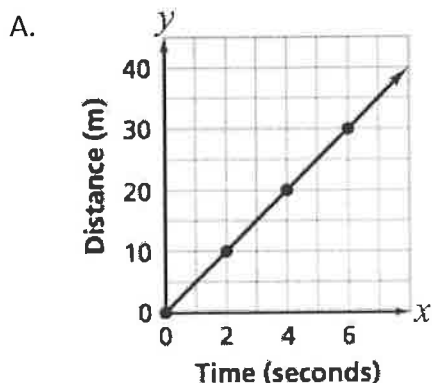
x	y
1	4
2	8
3	12
4	16

B.

x	y
3	12
5	20
2	8
8	32

- What is the constant of proportionality for table A? \_\_\_\_\_
- Write an equation of the form  $y = rx$  to represent table A. \_\_\_\_\_
- What is the constant of proportionality for table B? \_\_\_\_\_
- Write an equation of the form  $y = rx$  to represent table B. \_\_\_\_\_

2. The following graphs show a proportional relationship between x and y.



- What is the constant of proportionality for graph A? \_\_\_\_\_
- Write an equation of the form  $y = rx$  to represent graph A. \_\_\_\_\_
- What is the constant of proportionality for graph B? \_\_\_\_\_
- Write an equation of the form  $y = rx$  to represent graph B. \_\_\_\_\_

3. The following equations show a proportional relationship between x and y.

A.  $y = 3.7x$

B.  $y = \frac{4}{9}x$

- What is the constant of proportionality for equation A? \_\_\_\_\_
- What is the constant of proportionality for equation B? \_\_\_\_\_

# Constant of Proportionality and Proportional Equations (7.RP.2b, 7.RP.2c)

**Notes:**

- *constant of proportionality*: a constant ratio of two numbers
- Proportional equations should be of the form  $y = rx$ , where  $r$  is the constant of proportionality (unit rate)

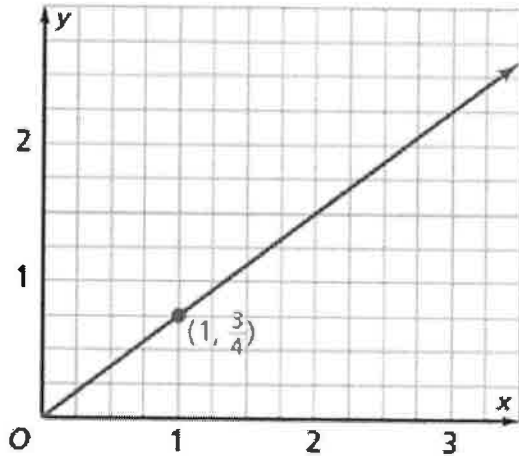
$$C = 13n$$

↑  
constant of proportionality

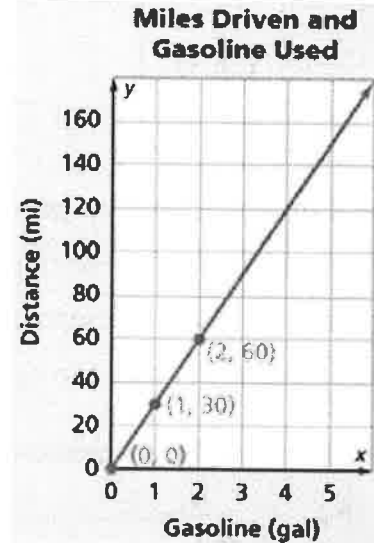
**Finding constant of proportionality (unit rate) from a TABLE:**

- Find a number that can multiply with every  $x$  value to get each corresponding  $y$  value
- Can set up a proportion or use division to calculate the constant of proportionality (unit rate)

**Examples:**



constant of proportionality:  $\frac{3}{4}$   
equation:  $y = \frac{3}{4}x$



constant of proportionality: 30  
equation:  $y = 30x$

**Finding constant of proportionality (unit rate) from a GRAPH:**

- Find the point  $(1, r)$ , where  $r$  is the constant of proportionality (unit rate) OR
- Can set up a proportion or use division to calculate the constant of proportionality (unit rate)

**Examples:**

$x$	$y$
4	$\cdot 12 = 48$
5	$\cdot 12 = 60$
8	$\cdot 12 = 96$

constant of proportionality: 12  
equation:  $y = 12x$

Time (hours)	8.25	6.25	9.25
Pages Lucas Read	208	156	234

constant of proportionality: 26  
equation:  $y = 26x$

**Finding constant of proportionality (unit rate) from an EQUATION:**

$$C = 13n$$

↑  
constant of proportionality

**Example:**

$y = \frac{7}{9}x$  constant of proportionality  $\frac{7}{9}$

**Explaining Coordinate Points (7.RP.2d)**

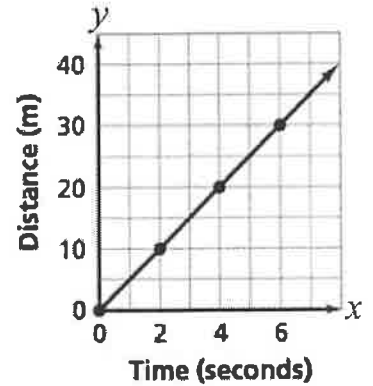
1. The graph at right shows a proportional relationship between  $x$  and  $y$ .

What do the following points represent in terms of the situation?

(0, 0)

(1, 5)

(6, 30)



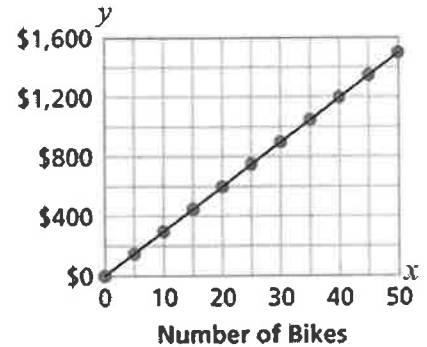
2. The graph at right shows a proportional relationship between  $x$  and  $y$ .

What do the following points represent in terms of the situation?

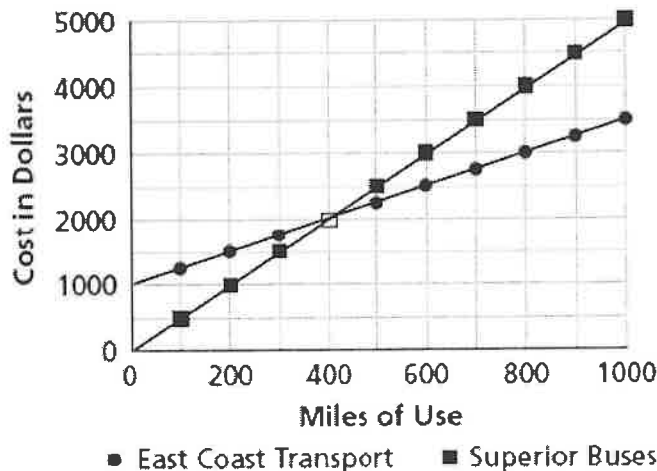
(0, 0)

(1, 30)

(40, 1200)



3. The following graph for Superior Buses shows a proportional relationship between  $x$  and  $y$ .

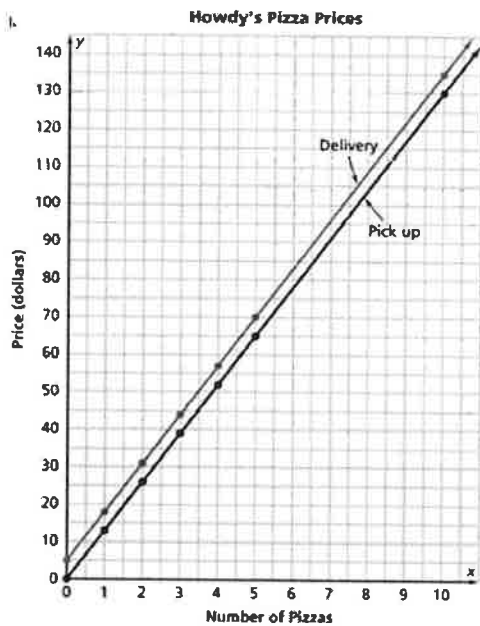


Select True or False for each statement about the graph.

Statement	True	False
Point $\square$ represents the total cost of travel when traveling for 400 miles.		
The total cost of travel is \$5 when traveling for 1 mile.		
The total cost of travel is \$600 when traveling for 3000 miles.		

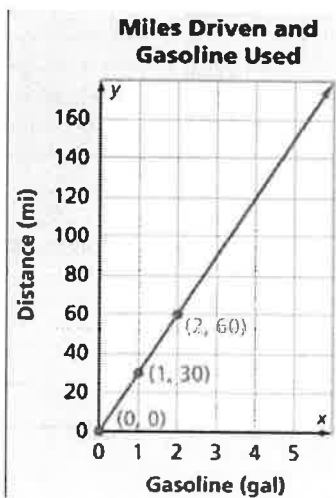
## Explaining Coordinate Points (7.RP.2d)

Examples:



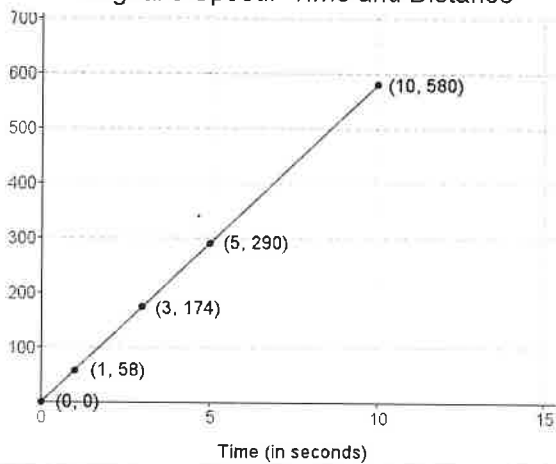
pick up  
 $(1, 13)$ : one pizza costs \$13  
 $(0, 0)$ : zero pizzas cost \$0

delivery  
 $(1, 18)$ : one pizza costs \$18  
 $(0, 5)$ : zero pizzas cost \$5



$(1, 30)$  = 1 gallon takes us 30 miles  
 $(0, 0)$  = 0 gallons takes us 0 miles

Jaguar's Speed: Time and Distance



What do the following points represent in the context of the situation?

- $(0, 0)$  A jaguar runs 0 feet in 0 seconds.
- $(1, 58)$  A jaguar runs 58 feet in 1 second.
- $(5, 290)$  A jaguar runs 290 feet in 5 seconds.



**Multi-Step Proportion Problems (7.RP.3)**

1. The ratio of blueberries to mango in a fruit smoothie drink is 5 to 15. What percent of the drink is mango? Write and solve a proportion to find the answer, and include units.
2. Bill has a paper route in his neighborhood. It takes him 45 minutes to deliver newspapers to the 30 customers on his route. How long will it take Bill to complete his route if he adds 25 more customers in his neighborhood? Write and solve a proportion to find the answer, and include units.
3. You have lunch at a Thai restaurant with your friends. The bill before tax is \$54.75, and the sales tax is 7%. You decide to leave a 20% tip for the waitress based on the pre-tax amount. What will be the amount of the total bill, including tax and tip? Write and solve proportions to find the answer, and include units.
4. Shirts Galore sells a Batman shirt for \$24.49, but is having a sale for 25% off. Rainbow Shirts sells the same Batman shirt for \$21.99, but is having a sale for 15% off. Which store offers the better price, after the discount? Write and solve proportions to find the answer, and include units with your answer.

## Multi-Step Proportion Problems (7.RP.3)

### Definition:

*proportion*: an equation stating that two ratios are equal.

### Examples:

Suppose that among American doctors, men outnumbered women by a ratio of 12 to 5. If about 600,000 American doctors are men, about how many are women? There are four ways to write this as a proportion:

Write the known ratio of men to women doctors. Complete the proportion with the ratio of actual numbers of doctors.

$$\frac{12 \text{ men}}{5 \text{ women}} = \frac{600,000 \text{ men}}{x \text{ women}}$$

Write a ratio of men to men data. Complete the proportion with women to women data.

$$\frac{12 \text{ men}}{600,000 \text{ men}} = \frac{5 \text{ women}}{x \text{ women}}$$

Write the known ratio of women to men doctors. Complete the proportion with the ratio of actual numbers of doctors.

$$\frac{5 \text{ women}}{12 \text{ men}} = \frac{x \text{ women}}{600,000 \text{ men}}$$

Write a different ratio of men to men data. Complete the proportion with women to women data.

$$\frac{600,000 \text{ men}}{12 \text{ men}} = \frac{x \text{ women}}{5 \text{ women}}$$

- Bill's bike shop has a sale where the bike shop pays the customer's tax. By law, Bill has to charge a 6% sales tax, so he finds a different way to take the tax off the bill. Bill decides to give each customer a 6% discount. Bill sells a bike that originally costs \$100.

Write and solve a proportion to find the cost with sales tax. Then, write and solve a proportion to find the amount of the discount. Then, calculate the final cost.

$$\frac{\$100}{100\%} = \frac{\$x}{106\%} \quad x = 100 \cdot 1.06 = \$106$$

$$\frac{\$106}{100\%} = \frac{\$x}{6\%} \quad x = 106 \cdot 0.06 = \$6.36$$

$$\begin{array}{r} \$106.00 \\ - \$6.36 \\ \hline \$99.64 \text{ final price} \end{array}$$

The Sports Depot is having a sale. Mike buys a pair of gloves that originally cost \$24, but are on sale for 25% off. He also buys a pair of skis that originally cost \$250, but are on sale for 15% off. What is the total amount that Mike pays for his new gear?

$$\frac{\$24}{100\%} = \frac{\$x}{25\%}$$

$$x = 24 \cdot 0.25 = \$6$$

$$24 - 6 = \$18 \text{ gloves}$$

$$\frac{\$250}{100\%} = \frac{\$x}{15\%}$$

$$x = 250 \cdot 0.15 = \$37.50$$

$$250 - 37.50 = \$212.50 \text{ skis}$$

$$18 + 212.50 = \$230.50 \text{ total}$$

**Factor and Expand Linear Expressions (7.EE.1)**

1. Simplify the following expressions completely.

a.  $10(1 + 4x) + 7(9x - 1)$

b.  $4(-3x - 6) + 6(1 - 9x)$

c.  $-9(1 + x) + 4(7x + 4)$

d.  $-6(9 - 3x) + 8(10 + 10x)$

e.  $-6(8 - 9x) + 8(6x + 10)$

f.  $9(1 - 9x) + 8(9x - 4)$

2. Use the Distributive Property to write ALL the possible expressions in factored form.

a.  $-4x + -8$

b.  $40x + 100$

c.  $40 + 32x$

d.  $100x - 20$

## Factor and Expand Linear Expressions (7.EE.1)

Notes:

You can use the **Distributive Property** to go between **factored form** and **expanded form**.

factored form

$$-3(4x + 8)$$

$$2(n - 6)$$

expanded form

$$= -12x + -24$$

$$= 2n - 12$$

expanding using Distributive Property

$$2(x + 6)$$

$$2 \cdot x + 2 \cdot 6$$

$$2x + 12$$

factoring using Distributive Property

$$20 + 4x$$

$$4 \cdot 5 + 4 \cdot x$$

$$4(5 + x)$$

### Combining Like Terms

**like terms:** have the same variable, and can be combined using addition and/or subtraction of the coefficients

\*\*It can help to highlight, underline, or box like terms.

<b>ex:</b> $y + 3x - 2 + 4y - 10x + 8 - x$ $-8x + 5y + 6$
--

Examples:

Use the **Distributive Property** to write each expression in **expanded form**.

1.  $3(4x + 6) = 12x + 18$

2.  $7(10x - 6) = 70x - 42$

3.  $-5(x + -2) = -5x + 10$

Use the **Distributive Property** to write each expression in **factored form**.

1.  $\frac{4x}{2} + \frac{2}{2} = 2(2x + 1)$

2.  $\frac{-6x}{2} + \frac{8}{2} = 2(-3x + 4)$

3.  $\frac{10x}{2} - \frac{6}{2} = 2(5x - 3)$

4.  $48x - 12$

$$2(24x - 6)$$

$$3(16x - 4)$$

$$4(12x - 3)$$

$$6(8x - 2)$$

$$12(4x - 1)$$

**Write and Explain Equivalent Expressions (7.EE.2)**

1. Gage goes to Burger Palace with 3 friends. Each person buys 3 cheeseburgers and 1 drink. Let  $C$  represent the cost of a cheeseburger and  $D$  represent the cost of a drink.

a. Using the distributive property, write TWO equivalent expressions that represent the total amount spent at the restaurant for all 4 people.

a. Explain how each expression describes the situation in a different way.

Expression 1:

Expression 2:

2. A Fitbit is on sale for 25% off the original price  $p$ .

a. Write TWO equivalent expressions that would find the final price of the Fitbit, from the original price  $p$ .

b. Explain how each expression describes the situation in a different way.

Expression 1:

Expression 2:

## Write and Explain Equivalent Expressions (7.EE.2)

### Notes:

equivalent expressions can be written using distributive property OR by combining like terms

### Examples:

**You and your friends made up a basketball game. Every shot made from the free throw line is worth 3 points and every shot made from the half-court mark is worth 6 points. 4 friends were playing this game.**

Using the distributive property, write TWO equivalent expressions that represent the total points scored. Let  $f$  represent free throw shots and  $h$  represent half court shots. Explain how each expression describes the situation in a different way.

$$4(3f + 6h)$$

Adds the number of 3-points shots to the number of 6-point shots, and is then multiplied by 4 people.

$$12f + 24h$$

Adds 12 points per 3-points shot for 4 people to 24 points per 6-point shot for 4 people.

An iPod is on sale for 35% off.

- a. Using the distributive property, write TWO equivalent expressions that would find the final price of the iPod, from the original price  $p$ .

①  $p - 0.35p$       ②  $0.65p$

- b. Explain how each expression describes the situation in a different way.

Expression 1:

this finds the 35% discount of the price, and subtracts it from the original price

Expression 2:

this finds the remaining 65% of the original price

**Write and Solve Equations (7.EE.4a)**

1. Sophia has a membership with an online music store. The membership costs \$19.99, and each song costs \$0.99. Sophia has saved \$118. How many songs can she buy?

Write an equation in terms of the number of songs  $n$ . Solve to find the answer and show all steps.

2. In winter, the price of apples suddenly went up by \$0.75 per pound. Sam bought 3 pounds of apples at the new price, for a total of \$5.88. What was the original price per pound?

Write an equation for the cost in terms of the original price per pound  $p$ . Solve to find the answer and show all steps.

3. Melissa's bank account has \$1250 dollars in it, and her internet bill is automatically deducting \$60 from her account every month. Her bank requires its customers to keep a minimum balance of \$350. If Melissa doesn't deposit any additional money in her account, after how many months will her account have only \$350?

Write an equation in terms of the number of months  $m$ . Solve to find the answer and show all steps.

4. Marco's grandma went on a vacation and asked Marco to take care of her tank of exotic fish. She promised to pay him \$3.50 for every fish in the tank when she returned. Four of the fish didn't survive, so Marco's grandma paid him \$59.50 for the remaining fish. How many fish were in the tank originally?

Write an equation in terms of the number of fish  $f$ . Solve to find the answer and show all steps.

## Write and Solve Equations (7.EE.4a)

### Steps:

- 1.) Use the Distributive Property to eliminate parentheses.
- 2.) Undo addition or subtraction.
- 3.) Undo multiplication or division.
- 4.) Check your answer.

Remember:

*The Golden Rule of Algebra*

*Do unto one side as you do to the other*

### Solving Examples:

$$\begin{array}{r} -7 + 3x = 23 \\ +7 \quad +7 \\ \hline 3x = 30 \\ \div 3 \quad \div 3 \\ \hline x = 10 \end{array}$$

$$\begin{array}{r} 18 + x/3 = 45 \\ -18 \quad -18 \\ \hline x/3 = 27 \\ \cdot 3 \quad \cdot 3 \\ \hline x = 81 \end{array}$$

$$\begin{array}{r} -160 = -8(3x + 8) \\ -160 = -24x + -64 \\ +64 \quad +64 \\ \hline -96 = -24x \\ \div -24 \quad \div -24 \\ \hline 4 = x \end{array}$$

### Writing Examples:

John has read the first 114 pages of a novel. He has read 3 pages less than one-third of the novel.

Write an equation to describe the total number of pages  $p$  in the novel. Then, solve algebraically to find the number of pages  $p$ .

$$\begin{array}{r} \frac{1}{3}p - 3 = 114 \\ +3 \quad +3 \\ \hline \frac{1}{3}p = 117 \\ \div \frac{1}{3} \quad \div \frac{1}{3} \\ \hline p = 351 \end{array}$$

The novel has 351 pages.

MacDonald had a farm of orange trees. He had to cut down 5 sick trees. Each of the remaining trees produced 210 oranges, producing a total harvest of 41,790 oranges.

Write an equation to represent the total harvest in terms of the initial number of trees  $t$  on MacDonald's farm. Then distribute and solve to find the initial number of trees  $t$ .

$$\begin{array}{r} 41790 = 210(t-5) \\ 41790 = 210t - 1050 \\ +1050 \quad +1050 \\ \hline 42840 = 210t \\ \div 210 \quad \div 210 \\ \hline 204 = t \end{array}$$

There were 204 orange trees initially

Jenny works in a local florist shop that sells roses for \$1.25 each, and charges \$0.50 to tie a ribbon around the bouquet. If the shop makes \$253, how many roses did they sell?

Write an equation for the total sales in terms of the number of roses sold,  $r$ . Solve to find the answer.

$$\begin{array}{r} 253 = 1.25r + 0.50 \\ -0.50 \quad -0.50 \\ \hline 252.5 = 1.25r \\ \div 1.25 \quad \div 1.25 \\ \hline 202 = r \end{array}$$

202 roses



**Write and Solve Inequalities (7.EE.4b)**

1. Tatiana wants to give friendship bracelets to her 32 classmates. She already has 5 bracelets, and she can buy more bracelets in packages of 4. How many packages does Tatiana need to buy?

a. Write an inequality in terms of the number of packages  $p$ . Solve to find the answer and show all steps.

b. Explain your answer in words. Then, graph the solution on a number line.



2. Renna pushed the button for the elevator to go up, but it would not move. The weight limit for the elevator is 450 kilograms, but the current group of passengers weighs a total of 750 kilograms. The passengers each weigh about 75 kilograms. How many passengers need to get off the elevator?

a. Write an inequality in terms of the number of passengers  $p$ . Solve to find the answer and show all steps.

b. Explain your answer in words. Then, graph the solution on a number line.



3. Joey is trying to break his own personal record, so he needs to eat more than 72 hot dogs in 10 minutes. After 1 minute of competition, Joey has eaten 10 hot dogs. How many hot dogs does he need to eat per minute for the remaining 9 minutes, in order to break his own record?

a. Write an inequality in terms of the number of hot dogs  $h$ . Solve to find the answer and show all steps.

b. Explain your answer in words. Then, graph the solution on a number line.



## Write and Solve Inequalities (7.EE.4b)

### Inequality Symbols and Graphing

Writing

< means: less than

> means: greater than

≤ means: less than or equal to

≥ means: greater than or equal to

Graphing

An open ○ means: NOT equal to < >

A closed ● means: equal to ≤ ≥

Then, draw an arrow showing all the possible solutions

### Solving Inequalities

you follow the same steps as solving equations!!!

- 1) Undo addition or subtraction.
- 2) Undo multiplication or division.
- 3) Check your answer.

However, there is one major difference...

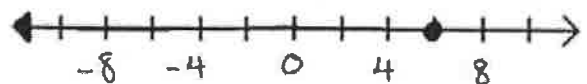
If you multiply or divide both sides by a NEGATIVE, then you must FLIP the inequality symbol.

### Solving and Graphing Examples:

$$\begin{array}{l}
 3. \quad -17 < -7x - 45 \quad -17 < -7(-5) - 45 \\
 \quad \quad \quad +45 \quad \quad \quad +45 \quad \quad \quad -17 < -10 \text{ true} \\
 \quad \quad \quad 28 < -7x \\
 \quad \quad \quad \div -7 \quad \div -7 \quad -17 < -7(-3) - 45 \\
 \quad \quad \quad \textcircled{-4 > x} \quad \quad \quad -17 < -24 \text{ false} \\
 \quad \quad \quad \textcircled{x < -4}
 \end{array}$$



$$\begin{array}{l}
 4. \quad \frac{x}{-2} \geq -3 \quad \frac{4}{-2} \geq -3 \\
 \quad \quad \quad \cdot -2 \quad \cdot -2 \quad \quad \quad -2 \geq -3 \text{ true} \\
 \quad \quad \quad \textcircled{x \leq 6} \\
 \quad \quad \quad \frac{8}{-2} \geq -3 \\
 \quad \quad \quad -4 \not\geq -3 \text{ false}
 \end{array}$$



### Writing Examples:

Ms. Chang's class decides to order posters that advertise the walkathon. Posters by Sue charges \$15 plus \$0.50 per poster.

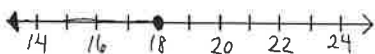
1. The class has a budget of \$24 for posters. What is the maximum number of posters they can order? Write an inequality for the cost in terms of number of posters  $p$  and solve to find the answer.

$$\begin{array}{r}
 0.5p + 15 \leq 24 \\
 \quad \quad \quad -15 \quad -15 \\
 \hline
 0.5p \leq 9 \\
 \div 0.5 \quad \div 0.5 \\
 \hline
 p \leq 18
 \end{array}$$

2. What does your answer mean in words?

The class can order 18 or less posters.

3. Graph the solution on a number line.



At Fabulous Fabian's Bakery, he makes a profit of \$4.95 per cake, but still has to deduct \$825 of expenses.

1. Fabian wants to make a profit of \$800 or more. What is the minimum number of cakes he needs to bake? Write an inequality for the profit in terms of number of cakes  $n$  and solve to find the answer.

$$\begin{array}{r}
 4.95n - 825 \geq 800 \\
 \quad \quad \quad + 825 \quad + 825 \\
 \hline
 4.95n \geq 1625 \\
 \div 4.95 \quad \div 4.95 \\
 \hline
 n \geq 328.28
 \end{array}$$

2. What does your answer mean in words?

He needs to bake 329 or more cakes.

3. Graph the solution on a number line.

