1. You can use properties of parallelograms to solve problems involving side lengths or angle measures of parallelograms. Quadrilaterals can be classified as parallelograms if they satisfy certain properties involving the opposite sides, the angles, or the diagonals. Parallelograms can be classified to be rhombuses, rectangles, or squares by properties of the diagonals.

2. square

3. kite

4. Interior

5. midsegment of a trapezoid

6. 540°, 360°; by the Polygon Interior Angles Theorem, interior angles of a n-sided polygon sum to $180^{\circ}(n-2)$. By the Polygon Exterior Angle-Sum Theorem, exterior angles of a convex polygon always sum to 360°. The figure has $n=5$ sides.

$$\begin{aligned} \theta &= 180^{\circ}(n-2) \\ &= 180^{\circ}(5-2) \\ &= 180^{\circ}(3) \\ &= 540^{\circ} \end{aligned}$$

7. 1800°, 360°; by the Polygon Interior Angles Theorem, interior angles of a n-sided polygon sum to $180^{\circ}(n-2)$. By the Polygon Exterior Angle-Sum Theorem, exterior angles of a convex polygon always sum to 360°. The figure has $n=12$ sides.

$$\begin{aligned} \theta &= 180^{\circ}(n-2) \\ &= 180^{\circ}(12-2) \\ &= 180^{\circ}(10) \\ &= 1800^{\circ} \end{aligned}$$

8. Yes; 90°, 135°, 135°, 90°, 135°, 135°; by the Polygon Interior Angles Theorem, interior angles of a n-sided polygon sum to $180^\circ(n-2)$. In the 6-sided polygon, there are four congruent angles and two right angles.

$$\begin{aligned}
&4(\theta) + 2(90^\circ) = 180^\circ(6-2)\\
&4\theta + 180^\circ = 180^\circ(4)\\
&4\theta + 180^\circ = 720^\circ\\
&4\theta = 540^\circ\\
&\theta = 135^\circ
\end{aligned}$$

9. 59°, by Theorem 6-3, the diagonals of a kite are perpendicular and, thus, intersect at right angles. Solve for $m\angle 1$.

$$\begin{aligned}
&31^\circ + m\angle 1 + 90^\circ = 180^\circ\\
&m\angle 1 = 59^\circ
\end{aligned}$$

10. 7 cm, by Theorem 6-6, the length of the midsegment is half the sum of the lengths of the bases. Solve for BC.

$$\begin{aligned}
&\tfrac{BC+13}{2} = 10\\
&BC + 13 = 20\\
&BC = 7
\end{aligned}$$

11. No, applying Theorem 6-6, the rod must be 42 in. long because $\frac{1}{2}(12+42) = 27$.

12. 85°; by Theorem 6-8, consecutive angles in a parallelogram are supplementary.

$$\begin{aligned}
&m\angle Z + m\angle W = 180^\circ\\
&(95^\circ) + m\angle W = 180^\circ\\
&m\angle W = 85^\circ
\end{aligned}$$

13. 95°; by Theorem 6-9, opposite angles in a parallelogram are congruent. $m\angle Z = m\angle X = 95^\circ$

14. He confused the lengths of the north and west sides; by Theorem 6-7, opposite sides of a parallelogram are congruent.

15. 12; applying Theorem 6-7, solve for x by setting the opposite sides of the quadrilateral equal.

$$\begin{aligned}
&3x + 4 = x + 28\\
&2x = 24\\
&x = 12
\end{aligned}
\quad \text{or} \quad
\begin{aligned}
&x + 3 = 2x - 9\\
&x = 12
\end{aligned}$$

16. 48; by Theorem 6-9, opposite angles in a parallelogram are congruent. Solve for x.

$$2x + 6 = x + 54$$
$$x = 48$$

17. Apply Theorem 6-15. If the length of each side along a vertical line is the same, then each quadrilateral is a parallelogram since vertical lines are parallel.

18. 6; by Theorem 6-16, the diagonals of a rhombus bisect each other, meet at right angles, and thus create four equal right triangles. Use $AD = 5$ and $EC = 4$ to solve for BD.

$$\begin{array}{ll} AE^2 + DE^2 = AD^2 & \\ 4^2 + DE^2 = 5^2 & BD = BE + DE \\ DE^2 = 25 - 16 & BD = 2DE \\ DE^2 = 9 & BD = 2(3) \\ DE = 3 & BD = 6 \end{array}$$

19. 12; applying Theorem 6-18, find QS.

$$\begin{aligned} QS &= PR \\ &= 2PT \\ &= 2(6) \\ &= 12 \end{aligned}$$

20. No, the quadrilateral could be a rhombus that is not a square.

21. Rectangle; the diagonals are congruent so by Theorem 6-21 the parallelogram is a rectangle.

22. Rhombus; the diagonals are perpendicular so by Theorem 6-19 the parallelogram is a rhombus.

23. 42; The diagram shows that $GHJK$ is a parallelogram. By Theorem 6-19, in order for $GHJK$ to be a rhombus, the diagonals must be perpendicular. Thus $48 + x = 90$, so $x = 42$.

24. No, the diagonals of both a kite and a rhombus are perpendicular, but if they bisect each other, the figure is a parallelogram, and, therefore, is a rhombus and not a kite.